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The Action of a Direct Radiator Loudspeaker with a Non-Linear Cone Suspension System

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During the past few years a number of mathematical investigators have directed their efforts toward the solution of differential equations with variable coefficients. These analyses are useful in explaining some of the phenomena which occur in electroacoustic vibrating systems with non-linear elements. In particular, this mathematics may be used to explain the various phenomena exhibited by a direct radiator loudspeaker with a non-linear cone suspension system. One of the effects is a jump phenomenon in the response frequency characteristic. Another effect is the production of harmonics and subharmonics.

INTRODUCTION

DURING the past few years a number of investigators[1] have directed their efforts toward the solution of differential equations with variable coefficients. These analyses are useful in explaining some of the phenomena which occur in electroacoustic vibrating systems with non-linear elements. In particular, this mathematics may be used to explain the various phenomena exhibited by a direct radiator loudspeaker with a non-linear cone suspension system.

The general trend in all types of radio receivers and phonographs is more output without a corresponding increase in the size of the loudspeaker. As a result, the maximum amplitude of the loudspeaker is also increased. Many apparently peculiar activities are manifested by the loudspeaker at the lower frequencies when the amplitude or excursion of the cone is large. Most of the unusual phenomena exhibited by the direct radiator loudspeaker at the lower frequencies are due to the non-linear characteristics of the suspension system. One of the effects of a non-linear

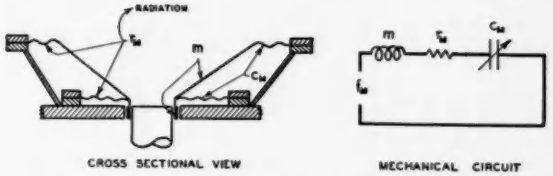

FIG. 1. Cross-sectional view of a dynamic type direct radiator loudspeaker and the mechanical circuit of the vibrating system. In the mechanical circuit: m, the mass of the cone and voice coil; C_M, the compliance of the suspension system; r_M, the mechanical resistance due to radiation and losses in the suspension system; f_M, the driving force.

[1] K. O. Friedrichs and J. J. Stoker, Quart. App. Math. 1, 97 (1943). This paper includes an extensive bibliography.

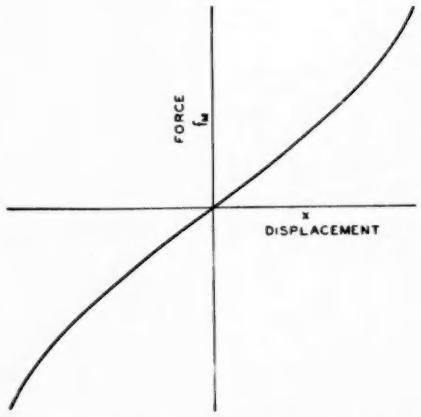

FIG. 2. The force displacement characteristic of the suspension system of the cone of a direct radiator loudspeaker.

cone suspension system is a jump phenomenon in the response characteristic. Another effect is the production of harmonics and subharmonics due to the non-linear cone suspension system. It is the purpose of this paper to demonstrate theoretically and experimentally the effect of a non-linear cone suspension system.

DIRECT RADIATOR LOUDSPEAKER

The simple direct radiator loudspeaker consists of a cone driven by voice coil located in a magnetic field. The essential mechanical elements and the mechanical circuit[2] of this loudspeaker for the low frequency range are shown in Fig. 1. The compliance or spring element in this system is the outside cone suspension and the voice coil centering suspension. In general, the compliance of the outside cone suspension is small compared to the compliance of the voice coil suspension and the cone suspension is therefore the controlling compliance. In any event, at the low frequencies the two compliances may be lumped as a single compliance as shown in the mechanical circuit of Fig. 1.

EQUATIONS DEPICTING THE ACTION OF A DIRECT RADIATOR LOUDSPEAKER WITH A NON-LINEAR SUSPENSION SYSTEM

The force displacement characteristic of a typical direct radiator loudspeaker cone suspension system is shown in Fig. 2. It will be seen that for small amplitudes the suspension system is linear. However, for large amplitudes the suspension system is non-linear.

The force deflection characteristic of the loudspeaker cone suspension system of Fig. 2 may be approximately represented by the expression

$$f_M = f(x) = \alpha x + \beta x^3, \tag{1}$$

where α = constant > 0, β = constant > 0, and f_M = applied force which produces the displacement x.

The compliance of the suspension system of Fig. 2 may be obtained from Eq. (1) as follows:

$$C_M = \frac{x}{f_M} = \frac{1}{\alpha + \beta x^2}. \tag{2}$$

The differential equation of the vibrating system in Fig. 1 is

$$m\ddot{x} + r_M\dot{x} + \frac{x}{C_M} = F\cos\omega t, \tag{3}$$

where x = displacement, $\dot{x}$ = velocity, $\ddot{x}$ = acceleration, m = mass of the cone and coil, r_M = mechanical resistance due to dissipation in the air load, C_M = compliance of the suspension system, $F = Bli$, B = magnetic flux density in the air gap, l = length of the voice coil conductor, i = amplitude of the current in the voice coil, $\omega = 2\pi f$, f = frequency, and t = time.

Substituting the expression for C_M of Eq. (2) in Eq. (3), the differential equation becomes

$$m\ddot{x} + r_M\dot{x} + \alpha x + \beta x^3 = F\cos\omega t. \tag{4}$$

FIG. 3. The wave shapes of the electrical input and the acoustical output illustrate the effect of a non-linear element upon the acoustical output.

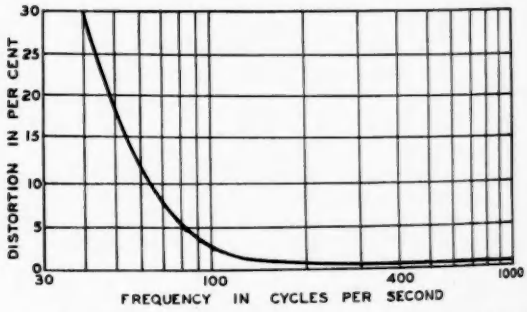

FIG. 4. Non-linear distortion frequency characteristic of an 8-inch dynamic type direct radiator loudspeaker mounted in a five-foot square baffle for an input of 5 watts.

[2] H. F. Olson, *Elements of Acoustical Engineering* (D. Van Nostrand Company, Inc., New York, 1940), Chapter VII.

Since the mechanical resistance r_M is quite small compared to the mechanical reactance, save over a very narrow frequency range near the resonance frequency, we can write Eq. (4) as follows:

$$m\ddot{x}+\alpha x+\beta x^3=F\cos\omega t. \tag{5}$$

A number of investigators have obtained an approximate solution of this differential equation. If β is considered to be small, the relation

$$\omega^2=\frac{\alpha}{m}+\frac{\frac{3}{4}\beta A^2}{m}-\frac{F}{Am} \tag{6}$$

between the arbitrary *amplitude* A and ω may be obtained.

An approximate solution of the differential equation, for unit mass, is

$$x=A\cos\omega t+\frac{1}{32}\frac{\beta A^3}{\alpha+\frac{3}{4}\beta A^2-(F/A)}\cos 3\omega t. \tag{7}$$

The sections which follow will show that these equations predict the performance of a loudspeaker with a non-linear cone suspension system.

NON-LINEAR DISTORTION CHARACTERISTICS

The well-known experimental result of a non-linear cone suspension system is the production of odd order harmonics when a sinusoidal input is applied to the loudspeaker. The wave shape under these conditions is shown in Fig. 3. The

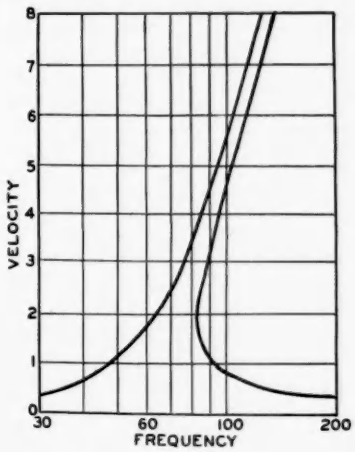

FIG. 5. Theoretical undamped response frequency characteristic of a direct radiator loudspeaker with a non-linear suspension system having a force displacement characteristic depicted in Fig. 2.

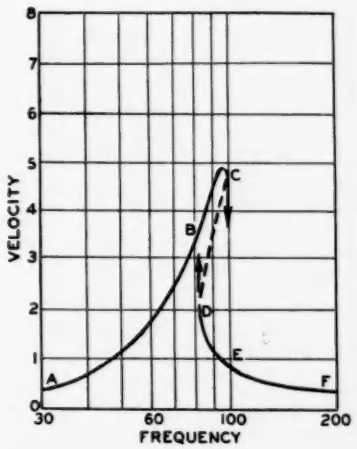

FIG. 6. Theoretical damped response frequency characteristic of a direct radiator loudspeaker with a non-linear suspension system. The dotted path represents a region of instability.

third harmonic is the preponderant distortion component. Equation (7) shows that a third harmonic term is introduced due to the suspension system. In the case of a direct radiator loudspeaker, the amplitude is inversely proportional to the square of the frequency for constant sound power output in the frequency region below the frequency of ultimate resistance. Consequently, the greatest distortion will occur at the low frequency end of the frequency range as shown by a typical experimental non-linear distortion frequency characteristic of Fig. 4. The manifestation and effect of this type of distortion upon the reproduction of sound are well known. It occurs in all amplifiers as well as loudspeakers. As a matter of fact, it is more troublesome in amplifiers because the distortion occurs over the entire audiofrequency range whereas the distortion is confined to the low frequency range in loudspeakers.

In the above considerations the distortion produced by the non-linear element comprises harmonics of the fundamental. Distortion components with frequencies of $1/2$, $1/3$, $1/4 \ldots 1/n$ of the frequency of the applied force also occur in non-linear systems. Those familiar with the performance of loudspeakers have noticed the production of subharmonics. In general, these are very pronounced in the mid-frequency range. In the mid-frequency range the subharmonics are due to the non-linear properties of the cone. Particular solutions of Eq. (3) have been ob-

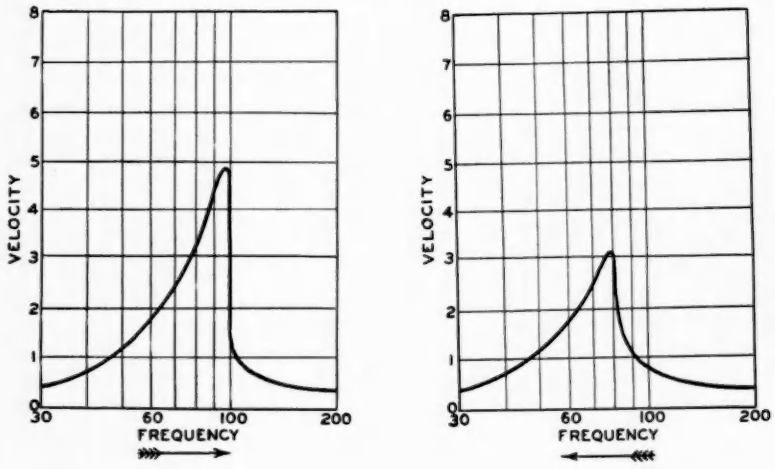

FIG. 7. Experimental response frequency characteristics of a direct radiator loudspeaker with a non-linear suspension system having a force displacement characteristic depicted in Fig. 2. A. (*left*) The response for an applied alternating voltage which continuously increases in frequency. B. (*right*) The response for an applied alternating voltage which continuously decreases in frequency.

tained which show that subharmonics are possible in a loudspeaker due to non-linear cone suspension system. As pointed out above, the amplitude of the cone of a direct radiator loudspeaker is inversely proportional to the square of the frequency for constant sound output. The large amplitudes are confined to the low frequency range. Therefore, these subharmonics will be of a very low frequency and difficult to detect. Careful experimental investigations have shown the existence of subharmonics due to a non-linear cone suspension system as predicted theoretically.

RESPONSE FREQUENCY CHARACTERISTICS

The velocity frequency characteristic of a loudspeaker with a non-linear suspension system may be obtained from the Eqs. (3) and (4). A typical theoretical velocity characteristic is shown in Fig. 5. In this analysis the mechanical resistance has been assumed to be zero. By analogy with the response curve in the linear case with damping one would expect the curve to be rounded off at the peak as shown in Fig. 6.

Suppose that we apply constant current to the voice coil of the loudspeaker and start at a low frequency A, Fig. 6. Then as we increase the frequency, the velocity increases steadily to the point C. At this point the velocity drops suddenly in a jump to point E. From point E on the velocity steadily decreases. Suppose that we now start at F and decrease the frequency. The velocity steadily increases to the point D. At point D the velocity suddenly jumps to the point B. From point B on the velocity steadily decreases.

Typical experimental velocity frequency characteristics are shown in Fig. 7. The velocity frequency characteristic for an increase in frequency is shown in Fig. 7A. The velocity frequency characteristic for a decrease in frequency is shown in Fig. 7B. These characteristics are quite similar to the theoretical characteristic of Fig. 6.

SUBJECTIVE ASPECTS OF SOUND REPRODUCTION BY A LOUDSPEAKER WITH A NON-LINEAR CONE SUSPENSION SYSTEM

The principal source of distortion due to a non-linear suspension system occurs in small table model and personal receivers, because in these receivers a relatively small cone is driven through relatively large amplitudes. Furthermore, to prevent damage to the cone and voice coil assembly the suspension is designed so that the amplitude of the cone is limited. When these systems reproduce speech and music at high levels non-linear distortion of the ordinary overload type is produced. There is in addition a peculiar type of distortion due to instability of the vibrating system. When a signal of varying frequency which includes the unstable frequency region is applied to the loudspeaker, extraneous frequencies are produced. The extraneous frequencies are manifested as grunting sounds. The loudspeaker jumps from one stable characteristic to the other with production of the extraneous sound or distortion during the transition period.

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The Effect of Radiation on the Vibrations of a Circular Diaphragm

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Circular plate with a fluid on one side: The free and forced vibration problems of a clamped plate in an infinite baffle are solved exactly in terms of a set of self- and mutual *radiation impedances* $Z_{mn}(\omega)$, associated with the normal modes of a plate vibrating in a vacuum. Computations of these impedances have been made for the lowest four modes over a sufficient range in frequency to solve the *forced vibration* problems that occur in practice as well as the *free vibration* problem of these modes. The magnitude of the influence of the fluid on the plate is shown to depend only on the frequency of vibration and the non-dimensional parameter $\beta=\rho_0 R/2\rho h$, where R is the radius of the plate, $2h$ the thickness, and ρ_0/ρ the relative density of fluid to plate. The *shift* in *resonant frequency*, and the *change in shape* of the lowest four modes due to the reaction of the fluid on the plate are computed as a function of β. These changes are found to be important in the range $1<\beta<8$. They are negligible for $\beta<1$, the case of radiation into air.

1. INTRODUCTION

THE presence of a fluid on one side of a vibrating plate has the following effects: (a) The plate pushes the fluid and thus appears to have an added inertia. This "mass loading" causes an appreciable lowering in the resonant frequencies of the plate. (b) The plate radiates acoustic energy into the fluid. This adds a resistive term to the impedances of the plate, and it causes a freely vibrating plate to decay in amplitude. (c) The fluid provides a coupling medium by means of which motion in one mode produces forces acting on another mode. This permits energy to be transferred from one mode to another. It causes the shape of the modes of a freely vibrating plate in contact with a fluid to be different from the corresponding modes in a vacuum.

This last effect can be visualized in the following manner. Suppose the plate is vibrating freely in its nth mode. If the pressure over the plate due to the fluid were proportional to the velocity distribution, then this shape of vibration would persist. In general, however, the distribution of pressure will not be the same as that of the vibration, and components of pressure will be exerted which tend to excite other modes. We may define $Z_{mn}(\omega)$ as the component of pressure acting on the mth mode when the plate is vibrating with frequency $\omega/2\pi$ and unit velocity in the nth mode. Then Z_{mn} will be a radiation impedance coupling the mth and nth modes. In the case $m=n$ this definition becomes what is usually termed the radiation impedance of the nth mode.

Horace Lamb,[1] in 1920, computed $Z_{11}(\omega)$, the radiation impedance of the first mode of a circular clamped plate in an infinite baffle. The resistive part of the impedance was obtained by assuming that the plate radiated spherically. This is true at low frequencies when the circumference of the plate is small compared with the wave-length. The reactive part, or mass loading, was found by assuming that the fluid is incompressible. This corresponds to assuming that the wave-length of sound in the fluid is infinitely large in comparison with the circumference of the plate, a condition which is exactly true for a compressible fluid only if the frequency is zero.

In this paper, the assumptions of incompressibility and spherical radiation will be dropped. The self- and mutual radiation impedances of the first mode and higher modes will be computed directly from the pressure distribution at the surface of a vibrating plate in an infinite baffle. The results will be applied to the free and forced vibration problems of such a plate.

2. FORMULATION OF THE PROBLEM

The equation of motion of a vibrating plate[2] when modified to take into account the reaction of a fluid on the plate is:

$$D\nabla^4 u+2\rho h\ddot{u}=G(r,t)-P(r,t), \qquad (2.1)$$

[1] H. Lamb, Proc. Roy. Soc. **A98**, 205 (1920).
[2] A. E. H. Love, *Mathematical Theory of Elasticity* (Cambridge University Press, London, 1934), fourth edition, Section 313.

where u is the displacement of the plate in the direction of the fluid, $G(r, t)$ is the applied pressure in the same direction, and $P(r, t)$ is the pressure exerted by the fluid on the plate at radius r and time t. The coefficients in (2.1) are defined by $D=\frac{2}{3}h^3E/(1-\sigma^2)$, where $2h$ is the thickness, ρ the density, E the Young's modulus, and σ the Poisson's ratio of the plate.

The fluid pressure reaction at (r, φ) on the surface of a plate vibrating in an infinite baffle is computed by regarding each element of area $dA'=r'dr'd\varphi'$ as a point source of sound and adding the resulting pressures.[3,4] For circularly symmetric vibrations this procedure gives:

$$P(r, t)=\frac{\rho_0}{2\pi}\int\int \ddot{u}(r', t-d/c)\frac{dA'}{d}, \quad (2.2)$$

where $d=\{r^2+r'^2-2rr'\cos(\varphi-\varphi')\}^{\frac{1}{2}}$ is the distance from the elemental source dA' to the point (r, φ) at which the pressure is being computed, ρ_0 is the density of the fluid, and c is the velocity of sound in the fluid.

We intend to solve the non-separable integro-differential equation formed by (2.1) and (2.2) by expanding the displacements and pressures in terms of the normal modes of a plate in a vacuum. This will lead to a set of linear equations whose coefficients turn out to be the radiation impedances discussed in the introduction. Before proceeding, we summarize briefly the properties of vibrating plates in a vacuum.[5]

3. NORMAL MODES IN A VACUUM

The shapes of the lowest four modes of a clamped circular plate are shown in Fig. 1. These are obtained from the general formula (3.1) for $u_n(r)$ which satisfies the proper boundary conditions for a clamped plate,[4] the free vibration equation (3.2) and the normalization conditions (3.3):

$$u_n(r)=2^{-\frac{1}{2}}\left[\frac{I_0(f_n y)}{I_0(f_n)}-\frac{J_0(f_n y)}{J_0(f_n)}\right], \quad (3.1)$$

where $y=r/R$.

$$D\nabla^4 u_n(r)-2\rho h\omega_n^2 u_n(r)=0, \quad (3.2)$$

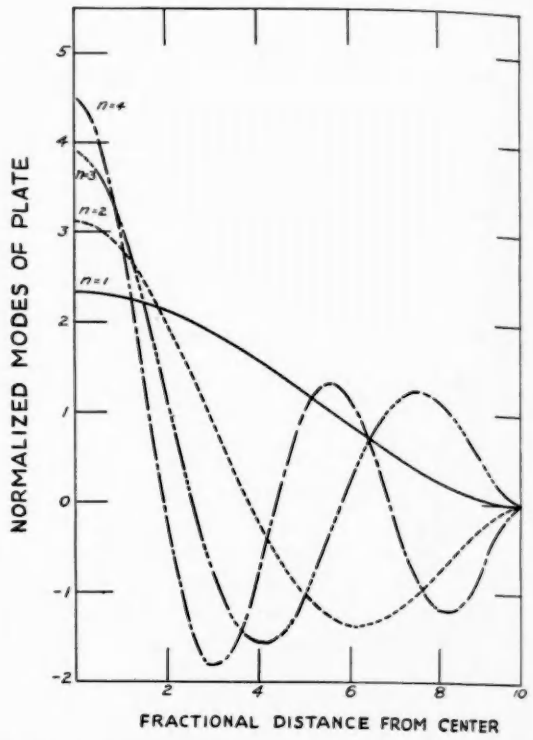

FIG. 1. The circularly symmetric normalized modes $u_n(y)$ of a plate of radius R vibrating in a vacuum are plotted as a function of the fractional distance from the center $y=r/R$. The first four modes are shown, modified by the factor $(-1)^{n+1}$ for convenience in plotting.

$$A^{-1}\int\int u_m(r)u_n(r)dA = 2\int_0^1 u_m(y)u_n(y)ydy=\delta_n{}^m, \quad (3.3)$$

where $A=\pi R^2$ is the area of the plate, $\delta_n{}^m$ is the Kronecker delta, and ω_n, the resonant frequency associated with the nth mode, is given by:

$$\omega_n=\frac{h}{R^2}\left\{\frac{E}{3\rho(1-\sigma^2)}\right\}^{\frac{1}{2}} f_n^2, \quad (3.4)$$

where f_n is the nth root of

$$I_0(f)J_1(f)+J_0(f)I_1(f)=0$$

and is given quite accurately[6] by $f_n\simeq n\pi$ for all n.

The average value of $u_n(r)$ over the plate is

$$\bar{u}_n=\frac{2^{\frac{1}{2}}}{f_n}\frac{I_1(f_n)}{I_0(f_n)}\xrightarrow[n\to\infty]{}\frac{2^{\frac{1}{2}}}{n\pi}, \quad (3.5)$$

[3] Lord Rayleigh, *Theory of Sound* (Macmillan and Company, London, 1937), second edition, Vol. 2, Section 278.
[4] P. M. Morse, *Vibrations and Sound* (McGraw-Hill Book Company, Inc., New York, 1936), first edition, p. 255.
[5] P. M. Morse, reference 4, p. 173.
[6] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1943), Alien Property Custodian edition, p. 234.

whereas the value at the center of the plate is

$$u_n(0)=2^{-\frac{1}{2}}\left[\frac{1}{I_0(f_n)}-\frac{1}{J_0(f_n)}\right]\rightarrow(-1)^{n+1}\pi(n/2)^{\frac{1}{2}}. \quad (3.6)$$

Numerical values are summarized in Table I with asymptotic formulas for use when $n>4$.

If the velocity of the plate for periodic vibrations is expressed in the form

$$\dot{u}(r,t)=\sum_{n=1}^{\infty}u_n(r)b_n\exp(j\omega t), \quad (3.7)$$

then b_n is defined to be the velocity of the nth mode. If the applied pressure

$$G(r,t)=G(r)\exp(j\omega t)$$

is expanded in a similar manner:

$$G(r,t)=\sum_{n=1}^{\infty}u_n(r)G_n\exp(j\omega t), \quad (3.8)$$

then only the nth term is effective in producing motion in the nth mode. The coefficient G_n is therefore referred to as the component of pressure acting on the nth mode. Making use of the orthonormality condition (3.3) it is given by:

$$G_n=A^{-1}\int\int G(r)u_n(r)dA. \quad (3.9)$$

With the definitions of component of pressure acting on a mode and velocity of a mode just given, the definition of radiation impedance, stated in the introduction, has been made precise and we may proceed to obtain an expression for this impedance.

4. RADIATION IMPEDANCES—GENERAL THEORY

When the velocity of the plate is given by (3.7) in terms of the normal modes of the plate, then the reaction of the fluid on the plate expressed by (2.2) becomes:

$$P(r,t)=\sum_{n=1}^{\infty}b_nP_n(r)\exp(j\omega t), \quad (4.1)$$

where $P_n(r)$, the pressure on the plate due to unit velocity in the nth mode, is

$$P_n(r)=\frac{j\omega\rho_0}{2\pi}\int\int u_n(r')\exp(-j\omega d/c)\frac{dA'}{d}. \quad (4.2)$$

What is the distribution of this pressure over the plate? Does it produce components of pressure which act on modes other than the nth? These questions can be answered by expanding $P_n(r)$ in terms of the normal modes of the plate:

$$P_n(r)=\sum_{m=1}^{\infty}Z_{mn}(\omega)u_m(r). \quad (4.3)$$

The coefficient Z_{mn} is by (3.8) the component of pressure acting on the mth mode of the plate due to $P_n(r)$, i.e., due to unit velocity in the nth mode. $Z_{mn}(\omega)$ by the definition given in the introduction is, therefore, the radiation impedance coupling the mth and nth modes. Using (3.9) we have:

$$Z_{mn}(\omega)=A^{-1}\int\int P_n(r)u_m(r)dA,$$

$$Z_{mn}(\omega)=\frac{j\omega\rho_0}{2\pi A}\int\int\int\int u_m(r)u_n(r')\times\frac{\exp(-j\omega d/c)}{d}dAdA'. \quad (4.4)$$

Introducing dimensionless variables $y=r/R$, $y'=r'/R$, $g=d/R$, $k=R\omega/c=2\pi R/\lambda$ the radiation integral becomes

$$\frac{Z_{mn}(k)}{\rho_0 c}=\frac{jk}{2\pi^2}\int\int\int\int u_m(y)u_n(y')\times\frac{\exp(-jkg)}{g}dSdS', \quad (4.5)$$

where

$$g^2=y^2+y'^2-2yy'\cos(\varphi-\varphi'),$$

$$dS=ydyd\varphi, \quad dS'=y'dy'd\varphi'.$$

The dimensionless radiation mass $m_{mn}(k)$, resistance $r_{mn}(k)$, and impedance $\alpha_{mn}(k)$ are defined by:

$$Z_{mn}/\rho_0 c=r_{mn}(k)+jkm_{mn}(k)=\alpha_{mn}(k) \quad (4.6)$$

TABLE I.

n	1	2	3	4	n (asymptotic)
f_n	3.19	6.306	9.425	12.56	$n\pi$
$\hat{u}_n$	0.741	0.411	0.293	0.224	$0.90/n$
$u_n(0)$	2.350	−3.134	3.901	−4.518	$(-1)^{n+1}2.221n^{\frac{1}{2}}$

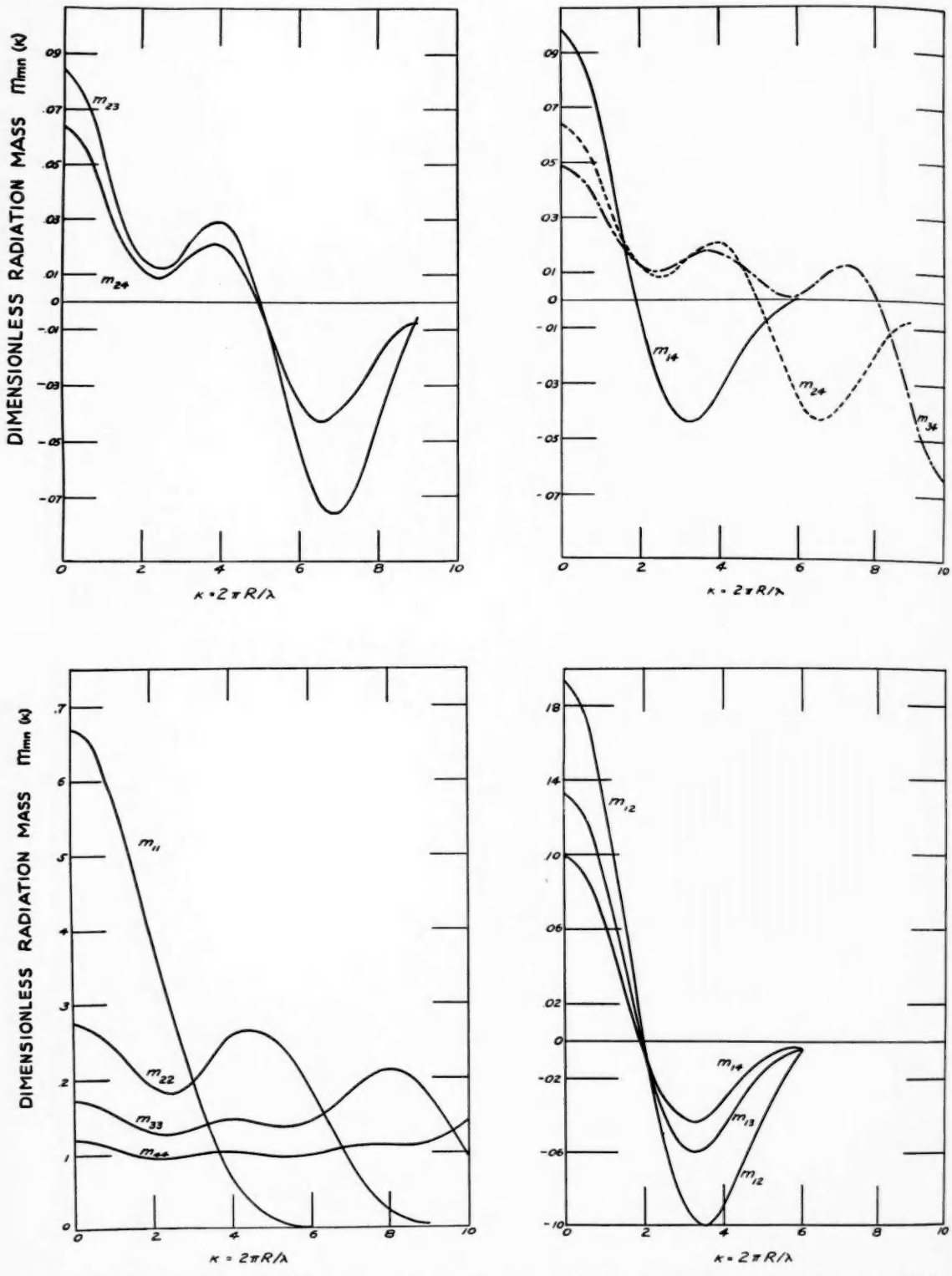

FIG. 2. The dimensionless radiation mass $m_{mn}(k)$ coupling the modes m and n of a circular clamped plate vibrating in an infinite baffle in contact with a fluid is plotted as a function of $k=2\pi R/\lambda$. The acoustic radiation impedance is related to the dimensionless mass by the equation $Z_{mn}=\rho_0 c[r_{mn}(k)+jkm_{mn}(k)]$, and the mass of fluid associated with modes m and n and vibrating with the plate is $\rho_0\pi R^2 m_{mn}(k)$.

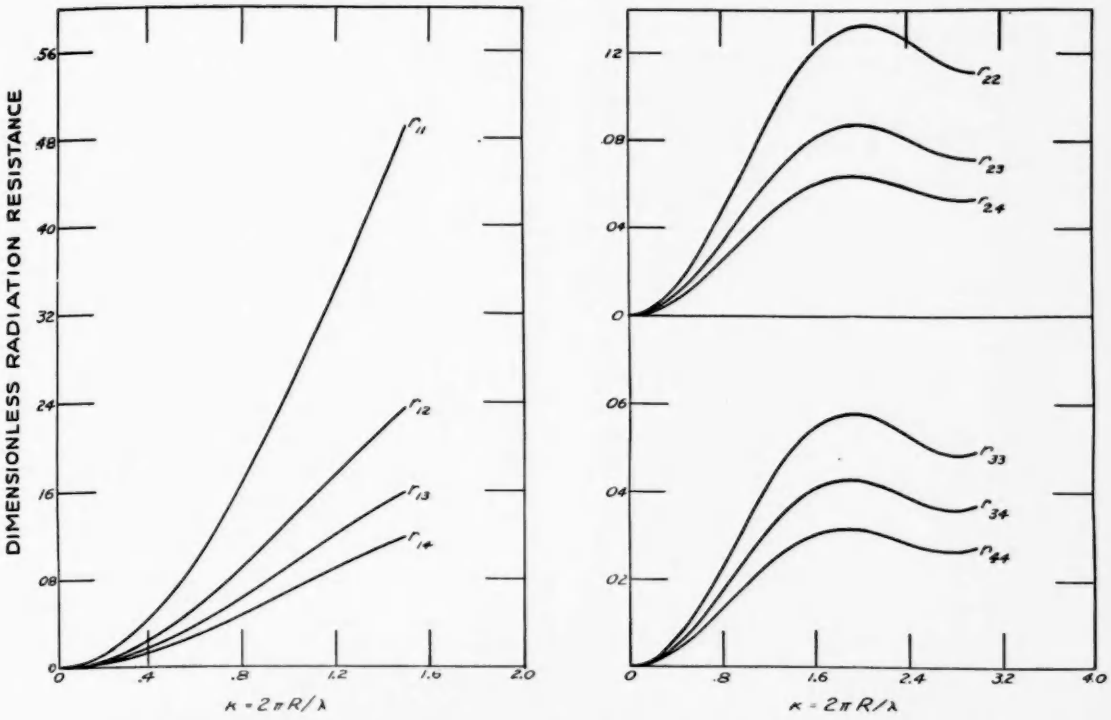

FIG. 3. The dimensionless radiation resistance $r_{mn}(k)$ coupling the modes m and n of a circular clamped plate vibrating in an infinite baffle in contact with a fluid is plotted as a function of $k=2\pi R/\lambda$. The acoustic radiation impedance is given in terms of the dimensionless resistance and mass by $Z_{mn}=\rho_0 c[r_{mn}(k)+jkm_{mn}(k)]$.

and numerical results are shown in Figs. 2 and 3. Since both resistance and mass loading are functions only of the dimensionless parameter $k=R\omega/c$, it is now obvious that Lamb's assumption of incompressibility $c=\infty$ was equivalent to evaluating the impedances at zero frequency.

Combining (4.1) and (4.3) the reaction of the fluid on the plate is expressible in the form:

$$P(r,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} b_m Z_{mn} u_n(r)\exp(j\omega t). \quad (4.7)$$

With this formula, and the curves shown in Figs. 2 and 3 it is possible for the reader to proceed directly to the section on free and forced vibrations.

5. EVALUATION OF THE IMPEDANCES

The chief difficulty in evaluating the radiation impedance integral (4.5) can be removed by means of the integral representation for a spherical wave in terms of cylindrical waves:[7, 8]

$$\exp(-jkg)/g=\int_0^{\infty}\lambda(\lambda^2-k^2)^{-\frac{1}{2}}J_0(\lambda g)d\lambda,$$
$$(\lambda^2-k^2)^{\frac{1}{2}}=j(k^2-\lambda^2)^{\frac{1}{2}} \quad \text{when} \quad \lambda<k \quad (5.1)$$

and the subsequent use of the addition theorem in cylindrical coordinates:[9]

$$J_0(\lambda g)=J_0(\lambda y)J_0(\lambda y')$$
$$+2\sum_{m=1}^{\infty}J_m(\lambda y)J_m(\lambda y')\cos m(\varphi-\varphi'). \quad (5.2)$$

Substituting these and integrating over φ and φ' causes all the terms in the series to vanish but the first. The integrations over y and y' can now be performed separately, and the dimensionless impedance $\alpha_{mn}(k)=Z_{mn}/\rho_0 c$ is expressed by the single integral:

$$\alpha_{mn}(k)=2jk\int_0^{\infty}\lambda(\lambda^2-k^2)^{-\frac{1}{2}}w_m(\lambda)w_n(\lambda)d\lambda, \quad (5.3)$$

[7] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), first edition, p. 576.
[8] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, London, 1922), p. 416.
[9] J. A. Stratton, reference 7, p. 373, Eq. (8).

where

$$w_n(\lambda)=\int_0^1 u_n(y)J_0(\lambda y)ydy$$

or

$$w_n(\lambda)=\frac{2^{\frac{1}{2}}f_n^{\,2}}{J_0(f_n)}\,\frac{J_0(f_n)\lambda J_1(\lambda)-f_nJ_1(f_n)J_0(\lambda)}{f_n^{\,4}-\lambda^4}. \quad (5.4)$$

The dimensionless radiation resistance $r_{mn}(k)$ and radiation mass $m_{mn}(k)$ defined by

$$Z_{mn}/\rho_0 c=\alpha_{mn}(k)=r_{mn}(k)+jkm_{mn}(k)$$

may then be obtained from:

$$m_{mn}(k)=2\int_k^\infty \lambda(\lambda^2-k^2)^{-\frac{1}{2}}w_m(\lambda)w_n(\lambda)d\lambda, \quad (5.5)$$

$$r_{mn}(k)=2k\int_0^k \lambda(k^2-\lambda^2)^{-\frac{1}{2}}w_m(\lambda)w_n(\lambda)d\lambda \quad (5.6)$$

by numerical integration. The resistance integral, however, simplifies for $k\ll\frac{1}{2}n\pi$ and $k\ll\frac{1}{2}m\pi$ for then it is possible to neglect λ^4 in comparison with β_n^4 in $w_n(\lambda)$ or β_m^4 in $w_m(\lambda)$. The simplified integral can be evaluated with the result:

$$r_{mn}(k)\cong\tfrac{1}{2}\bar{u}_m\bar{u}_nk[A(k)-(a_m+a_n)\times B(k)+a_ma_nC(k)], \quad (5.7)$$

where $\bar{u}_n$ is given by (3.5), $a_n=J_0(f_n)/f_nJ_1(f_n)$; and

$$A(k)=\tfrac{1}{2}\int_0^{2k}J_0(t)dt,$$

$$B(k)=0.5A(k)-0.5kJ_0(2k),$$

$$C(k)=[0.5A(k)-0.75J_1(2k)]k^2+0.75B(k).$$

Figure 2 showing $m_{mn}(k)$ was obtained by numerical integration of (5.5) with suitable modifications to eliminate the infinity at $\lambda=k$. Figure 3 is based on (5.7) all of whose terms, including $A(k)$, are expressible by means of tabulated functions.[10]

The properties of the radiation impedances at low and high frequencies obtained directly from the integrals are summarized as follows:

$$m_{nn}(0)\simeq 0.43/n+0.24/n^2 \qquad n\geqq 1, \quad (5.8a)$$

$$r_{mn}(k)\simeq\tfrac{1}{2}\bar{u}_m\bar{u}_nk^2 \qquad \text{as } k\to 0, \quad (5.8b)$$

$$m_{mn}(k)=O(k^{-6}) \qquad \text{as } k\to\infty, \quad (5.8c)$$

$$r_{mn}(k)=\delta_n{}^m+O(k^{-2}) \qquad \text{as } k\to\infty, \quad (5.8d)$$

$$m_{nn}(k)\simeq m_{nn}(0) \quad \text{for large } n \quad \text{if } k\ll n\pi. \quad (5.8e)$$

The form of property (a) comes from consideration of the integral, and the constants are obtained empirically from the values of $m_{nn}(0)$ for $n=1, 2, 3, 4$. Property (b) is the first term in the power expansion of $r_{mn}(k)$. In the case $m=n$, the same result would be obtained from the volume displacement $A\bar{u}_n$ if the plate vibrating in its nth mode is considered to radiate energy as a simple source.[11] This was the procedure used by Lamb is obtaining the radiation resistance of the first mode.[1]

The fact that the radiation resistances of the modes approach unity at infinite frequency, whereas the coupling resistances approach zero, follows from the orthogonality of the set of functions $w_n(\lambda)$ with respect to the weight function λ. This is true because by (5.3) $w_n(\lambda)$ is the Bessel transform of $u_n(y)$ and $u_n(y)$ satisfies the orthonormality conditions (3.3).

Property (e) states that the functions $m_{nn}(k)$ approach a constant for large n and fixed $k\ll n\pi$. This is a consequence of the fact that the major contribution to the integral (4.12) occurs near $\lambda=f_n\simeq n\pi$.

These properties, in conjunction with Figs. 2 and 3, may now be used to investigate the motion of a plate in contact with a fluid.

6. FREE AND FORCED VIBRATIONS

The equation of motion of a plate was stated in (2.1). If all the quantities are expressed in terms of the normal modes of the plate by means of (3.7), (3.8), (3.2), and (4.7) and coefficients of $u_n(r)$ on each side of (2.1) are equated, a set of linear equations is obtained:

$$2\rho h(\omega_n^{\,2}/j\omega+j\omega)b_n=G_n-\sum_{m=1}^{\infty}b_mZ_{mn}(\omega). \quad (6.1)$$

If dimensionless variables are introduced in the notation of (4.5) and (4.6) with $k_n=R\omega_n/c$,

[10] A. N. Lowan and M. Abramowitz, J. Math. and Phys. **22**, 2 (1943).

[11] P. M. Morse, reference 4, p. 242.

then (6.1) becomes:

$$b_n[k_n^2/jk+jk+\beta\alpha_{nn}(k)] = \frac{R}{2\rho hc}G_n-\beta\sum{}' b_m\alpha_{mn}, \quad (6.2)$$

where $\sum'$ is a summation over all modes but the nth and the dimensionless parameter $\beta=\rho_0 R/2\rho h$ determines the importance of the radiation terms in (6.2). For small β the plate behaves as if it were in a vacuum, whereas if $\beta>1$ the influence of the medium on the motion of the plate is appreciable.

To solve a given forced vibration problem one must first obtain the components of pressure G_n from (3.9) and then compute the velocities b_n from (6.2).

For a uniform applied force $G_n=\bar{u}_n\bar{G}$ and for a force concentrated at the center $G_n=u_n(0)\bar{G}$, where $\bar{G}$ is the average applied pressure defined by $\bar{G}=A^{-1}\int\int G(r)dA$. The quantities $\bar{u}_n$ and $u_n(0)$ are given in Section 3.

The linear equations (6.2) can be solved by means of a rapidly converging successive approximation procedure.[12] A good approximation may be obtained at the start by:

$$b_n\simeq\frac{R}{2\rho hc}\frac{G_n}{[k_n^2/jk+jk+\beta\alpha_{nn}(k)]}, \quad (6.3)$$

which takes into account the radiation mass and resistance of the nth mode but neglects coupling with other modes. A better approximation may be obtained by substituting these results into the right number of (6.2) and solving again for b_n. Repeating this procedure will usually cause little change in the results.

The energy radiated per unit area, per second can then be computed from:

$$I=\tfrac{1}{4}\rho_0 c\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} r_{mn}(k)[b_m\bar{b}_n+b_n\bar{b}_m], \quad (6.4)$$

where the bars represent complex conjugates. For motion *only* in the nth mode, this reduces to the classical result, $I=\frac{1}{2}\rho_0 c r_{nn}|b_n|^2$.

The free vibration problem is similar to the forced vibration problem in that a solution of the linear equations (6.2) is required. In this case, however, $G_n=0$ and we have homogeneous linear equations which possess non-vanishing solutions only for certain values of k, namely, the new resonant frequencies of the plate. Both these and the coefficients b_s can be determined by successive approximation procedures. In the case of the nth mode, the nth equation of (6.2) should be used to solve for k_n/k, the ratio of old to new resonant frequencies, and the sth equation should be used in conjunction with the value of k just determined to obtain ${}_nb_s$. Neglecting radiation

[12] R. V. Southwell, *Relaxation Methods in Engineering Science* (Oxford, Clarendon Press, London, 1940).

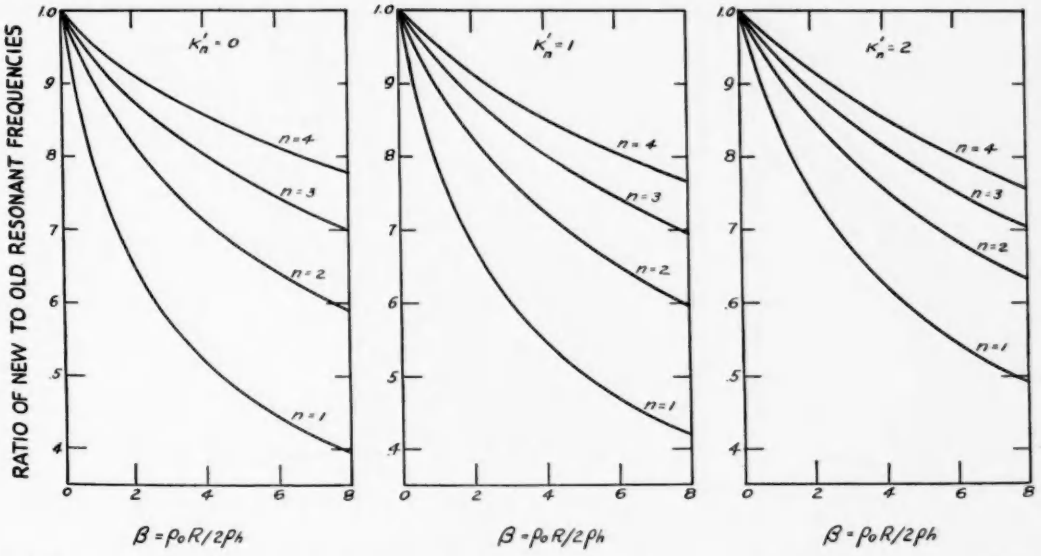

FIG. 4. The ratio of new to old resonant frequencies k_n'/k_n of the nth mode of the plate is plotted as a function of $\beta=\rho_0 R/2\rho h$ for values of the new resonant frequency $k_n'=k=0, 1, 2$ where $k=2\pi R/\lambda=R\omega/c$.

resistances at first, this procedure gives:

$$(k_n/k)^2=1+\beta m_{nn}(k)+\beta\sum{}' {}_nb_m m_{mn}(k), \quad (6.5)$$

$${}_nb_s=\frac{\beta m_{sn}(k)}{(k_s/k)^2-(1+\beta m_{ss})} - \frac{\beta\sum{}'' {}_nb_m m_{sm}(k)}{(k_s/k)^2-(1+\beta m_{ss})}, \quad (6.6)$$

where the major term in (6.6) has been separated from the others, and $\sum''$ means a summation over m which does not include s or n. Also, ${}_nb_n$ has been set equal to unity, so that the magnitude of the coefficients ${}_nb_s$, $s\neq n$ in the series (3.7)

$$\dot{u}(r,t)=\sum_{s=1}^{\infty} {}_nb_s u_s(r)\exp(j\omega_n' t)=U_n(r)\exp(j\omega_n' t)$$

indicates the change in shape of the nth mode of the plate. $\omega_n'=ck_n'/R$ is the new resonant frequency of the plate, where k_n' is the value of k determined by the procedure just described, and $U_n(r)$ is the new shape of the nth mode.

In practice, it was found simpler to specify k and regard k_n as variable, using (6.5) to determine k_n/k. In this manner Fig. 4 and Table II were constructed giving the ratio of new to old resonant frequencies k_n'/k_n and the associated coefficients ${}_nb_s$, for the first four modes, for $k=0$, 1, 2 and over the range $\beta=0$ to 8. One may therefore calculate $\beta=\rho_0 R/2\rho h$ and k_n from (3.4) and enter Fig. 4 to obtain k_n'/k_n. The error caused by using k_n rather than k_n' in entering the table can be corrected once an approximate value for k_n' is found.

To illustrate the changes which take place in a successive approximation solution of (6.5), consider the case of the second mode of a plate such that $\beta=\rho_0 R/2\rho h=5$ and $k=R\omega/c=1$. For an incompressible fluid, Lamb's method of approximation gives:[1] $(k_2/k)^2=1+5m_{22}(0)=2.385$. If the dependence of $m_{22}(k)$ on frequency is taken into account: $(k_2/k)^2=1+5m_{22}(1)=2.225$ and if the coupling terms in (6.4) are used, the result is $(k_2/k)^2=2.144$. The effect of dissipation on the square of the resonant frequency which has been neglected in (6.2) is to reduce it by the factor $(1-\frac{1}{4}Q^{-2})$ a change of 0.6 percent since the Q of the second mode is in this case $Q_2\simeq 6.6$.

The effect of radiation on resonant frequency is negligible, but its effect on the Q's and decay times of the modes is appreciable. These are given approximately by:

$$Q_n\simeq k[1+\beta m_{nn}(k)]/[\beta r_{nn}(k)], \qquad \tau_n\simeq 2\frac{R}{c}\frac{Q_n}{k}, \quad (6.7)$$

where k is to be set equal to k_n' the new resonant frequency of the nth mode.

The calculations made in this free vibration problem are good to about 1 percent. Lamb's result for the first mode[1] $(k_1'/k_1)=(1+0.6689\beta)^{-\frac{1}{2}}$ can be compared with ours providing the coupling terms in (6.5) are neglected, and the incompressibility assumption $k=0$ is used. This gives $(k_1'/k_1)=[1+m_{11}(0)\beta]^{-\frac{1}{2}}$, where $m_{11}(0)=0.670$. The difference between the two results is negligible.

TABLE II. Values of ${}_nb_s(k,\beta)$. The new shape of the nth mode of the plate is given by $U_n(r)=\sum_{s=1}^{\infty} {}_nb_s u_s(r)$. By definition ${}_nb_n=1$ and, if $\beta=0$, ${}_nb_s=\delta_s{}^n$. The other values are given below.

(k,β)	${}_1b_2$	${}_1b_3$	${}_1b_4$	${}_2b_1$	${}_2b_3$	${}_2b_4$
0.2	0.0492	0.0134	0.0054	−0.197	0.0789	0.0257
0.4	.0609	.0168	.0069	−.244	.111	.0368
0.6	.0661	.0184	.0075	−.265	.128	.0430
0.8	.0691	.0192	.0079	−.276	.140	.0470
1.2	.0342	.0094	.0037	−.137	.0560	.0179
1.4	.0431	.0120	.0047	−.172	.0806	.0262
1.6	.0472	.0132	.0052	−.189	.0944	.0309
1.8	.0496	.0140	.0055	−.199	.104	.0340
2.2	−.0012	−.0009	−.0006	.0048	.0178	.0056
2.4	−.0016	−.0012	−.0009	.0065	.0269	.0086
2.6	−.0019	−.0014	−.0009	.0074	.0324	.0104
2.8	−.0020	−.0015	−.0010	.0079	.0362	.0117

	${}_3b_1$	${}_3b_2$	${}_3b_4$	${}_4b_1$	${}_4b_2$	${}_4b_3$
0.2	−0.121	−0.178	0.0841	−0.087	−0.103	−0.150
0.4	−.151	−.250	.127	−.110	−.147	−.226
0.6	−.165	−.290	.153	−.120	−.172	−.272
0.8	−.173	−.315	.171	−.126	−.188	−.303
1.2	−.0844	−.126	.0594	−.0588	−.0715	−.106
1.4	−.108	−.181	.0916	−.0758	−.105	−.163
1.6	−.119	−.212	.112	−.0838	−.124	−.199
1.8	−.126	−.233	.126	−.0885	−.136	−.223
2.2	.0081	−.0400	.0232	.0093	−.0224	−.0409
2.4	.0111	−.0605	.0366	.0128	−.0343	−.0651
2.6	.0127	−.0730	.0457	.0147	−.0417	−.0812
2.8	.0136	−.0814	.0521	.0158	−.0467	−.0926

7. CONCLUSION

The free and forced vibration problems of a clamped plate in an infinite baffle were formulated exactly in Section 2. The reaction of the fluid on a plate vibrating with an arbitrary velocity distribution was obtained in (4.7). These results

were combined in Section 6 and the problem was reduced to the solution of a set of linear equations (6.1) whose coefficients are the radiation impedances of the plate.

These impedances were simplified to a single integration in Section 5, and a discussion of their low and high frequency properties was given. The radiation masses and resistances of the first four modes are plotted in Figs. 2 and 3 as a function of $k=2\pi R/\lambda$. The only approximation made in obtaining these impedances occurred in the evaluation of the integrals.

The influence of the fluid on the motion of the plate was shown by (6.2) to depend primarily on the dimensionless parameter $\beta=\rho_0 R/2\rho h$. The ratio of new to old resonant frequencies k_n'/k_n was computed for the first four modes for $\beta=0$ to 8 and $k_n'=0, 1, 2$. Numerical results are summarized in Fig. 4. The new shape of the nth mode was expressed in the form

$$U_n(r)=\sum_{s=1}^{\infty} {}_nb_s u_s(r),$$

where ${}_nb_n=1$ and the other coefficients ${}_nb_s$ are a measure of the change in shape of the nth mode. These coefficients are given in Table II for the first four modes, over the same range in β and k.

The procedures developed in this paper can be extended or applied in several ways: (1) Radiation impedances can be obtained for modes with nodal diameters. (2) Membranes, strings, and antennas may be handled in a similar manner. [In particular (4.7) and (5.3) apply to membranes as well as plates if $u_n(r)$ is regarded as the nth mode of a membrane.] (3) The radiation impedances can be used to solve transient problems by making use of the Laplace or Fourier transforms.

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The Reflection of Sound due to a Change in Cross Section of a Circular Tube

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The reflection and transmission coefficients due to a change in cross section of a circular tube are determined by calculating the exact pressure distribution in the vicinity of the discontinuity. The fluid medium in the tube is assumed to be ideal in the sense that the pressure distribution satisfies the wave equation. In contrast to the simpler theory which assumes a tube which is small compared to the wave-length, the following theory is subjected only to the assumption of an ideal fluid.

THE usual theory of constrictions and obstacles in a tube in which sound is being propagated assumes either that the dimensions of the discontinuity are very small or very large compared to the wave-length.[1,2] In the case of a change of cross section of a circular tube it is generally assumed that the ratio of diameter to wave-length is small enough so that only plane wave propagation need be considered.[1,3] Experimental verification of a theory based on this assumption is good at low frequencies, but breaks down at higher frequencies as should be expected.[1] The theory developed below is valid for any frequency (in an ideal fluid), but is considerably more complicated.

We first determine the pressure distribution in an infinite circular tube of radius a. If the fluid medium within the tube is assumed to be ideal it can be shown[2] that the pressure* distribution within the tube must satisfy the well-known wave equation

$$\nabla^2 p - \frac{1}{C^2}\frac{\partial^2 p}{\partial t^2} = 0, \tag{1}$$

where C is the velocity of sound in the medium. If we assume the pressure to be a sinusoidal function of time, (1) can be satisfied by writing

$$p = U(u_1, u_2) \exp\left\{ j\left[Wt \pm (W^2 - W_{mn}^2)^{\frac{1}{2}} \frac{z}{C} \right] \right\}, \tag{2}$$

which is the equation of a pressure wave being propagated along the z axis of a cylindrical tube, the coordinates of whose cross section are u_1 and u_2, and $U(u_1, u_2)$ satisfies the equation.

$$\nabla^2 U + (W_{mn}^2/C^2)U = 0. \tag{3}$$

The wave is being propagated in the positive or negative Z direction, according as the sign in the exponential of (2) is negative or positive, with a phase velocity

$$V_{mn} = CW(W^2 - W_{mn}^2)^{-\frac{1}{2}}, \tag{4}$$

where W_{mn} are the eigenvalues to be determined by the boundary conditions. By applying Newton's law (neglecting the gravitational field) it can be shown[2] that the velocity at any point in the fluid is given by

$$\bar{q} = (1/jW\rho_0)\nabla p, \tag{5}$$

where ρ_0 is the density of the fluid at rest.

The solution to (3) in polar coordinates is known to be[4]

$$U(r, \theta) = [AJ_m(W_{mn}r/C) + BY_m(W_{mn}r/C)] \cos(m\theta + \beta m), \tag{6}$$

unless $W_{mn} = 0$, in which case (3) reduces to Laplace's equation having the solutions

$$U = \text{const.}, \tag{7}$$

$$U = (A \log r + B)(C\theta + D), \tag{8}$$

$$U = (Er^p + Fr^{-p}) \cos(p\theta + \beta_p). \tag{9}$$

In the case of a circular tube of radius a the pressure must remain finite at $r = 0$ so that in

[1] G. W. Stewart and P. B. Lindsay, *Acoustics* (D. Van Nostrand Company, Inc., New York, 1930).
[2] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1936).
[3] H. E. Hartig and C. E. Swanson, Phys. Rev. **54**, 618–628 (1938).
* I.e., the pressure in excess of the static pressure; the use of Eq. (1) also implicitly assumes that this excess pressure is small compared to the static pressure.
[4] See, for instance, W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Company, Inc., New York, 1939).

(6) $B=0$, in (9) $F=0$, and the logarithmic solution of (8) is ruled out. Moreover, the pressure must be a single-valued function of position so that m and p must be integers. The boundary conditions demand that the radial component of the velocity as given by (5) must vanish identically at $r=a$. Since $F=0$, this boundary condition now demands that $E=0$ unless $p=0$, in which case (9) reduces to (7). Setting the derivative of (6) equal to zero at $r=a$ we obtain the characteristic equation for W_{mn}

$$J_m'(W_{mn}a/C)=0, \tag{10}$$

where the prime denotes differentiation with respect to the entire argument. There will be an infinity of roots to (10) which we denote by $n=0, 1, 2, 3, \cdots \infty$. The complete solution for the pressure distribution now becomes

$$\begin{aligned} p=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} & P_{mn}J_m(W_{mn}r/C)\cos(m\theta+\beta_m) \\ & \times\exp\{jW[t\pm(z/V_{mn})]\} \\ & +P_{00}\exp\{jW[t\pm(z/C)]\}, \end{aligned} \tag{11}$$

which can also be written

$$\begin{aligned} p=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} & J_m(W_{mn}r/C) \\ & \times[P_{mn}\cos m\theta+Q_{mn}\sin m\theta] \\ & \times\exp\{jW[t\pm(z/V_{mn})]\} \\ & +P_{00}\exp\{jW[t\pm(z/C)]\}, \end{aligned} \tag{12}$$

since $n=0$ implies $W_{mn}=0$ so that (7) must be used. We see that (7) defines a plane wave traveling with a velocity C, while each term in the series of (11) represents a mode traveling with a phase velocity V_{mn}. If W is greater than W_{mn} then V_{mn} is real and we have a propagated mode, but if W is less than W_{mn}, V_{mn} is imaginary, and we have a reflected or attenuated mode (the sign of the exponent must be chosen to insure attenuation if V_{mn} is imaginary). The phase velocity, when real, is always greater than C, but it is not the velocity with which a signal is propagated. The signal (or "group") velocity is given by the relation[5]

$$V_s=C^2V_{mn}^{-1}. \tag{13}$$

[5] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941).

Since V_{mn} is a function of the frequency there is danger of a modulated signal breaking up if it is transmitted too far in a pipe which propagates more than one mode; i.e., phase distortion may take place.

Due to Eq. (10) and the properties of the Bessel and trigonometric functions, (11) represents an orthogonal expansion, so that the coefficients P_{mn} may be determined by the usual means.[6] In particular if the pressure distribution at $z=0$ is given by

$$p=\bar{p}(r,\theta) \tag{14}$$

we can obtain[7]

$$\begin{aligned} P_{mn}&=\frac{2\left[a^2-\dfrac{m^2c^2}{W_{mn}^2}\right]^{-1}}{\pi J_m^{\,2}\left(\dfrac{W_{mn}a}{C}\right)}\int_0^a\int_0^{2\pi}\bar{p}(r,\theta) \\ &\qquad\times J_m\left(\frac{W_{mn}r}{C}\right)\cos m\theta r\,d\theta\,dr, \\ Q_{mn}&=\frac{2\left[a^2-\dfrac{m^2c^2}{W_{mn}^2}\right]^{-1}}{\pi J_m^{\,2}\left(\dfrac{W_{mn}a}{C}\right)}\int_0^a\int_0^{2\pi}\bar{p}(r,\theta) \\ &\qquad\times J_m\left(\frac{W_{mn}r}{C}\right)\sin m\theta r\,d\theta\,dr, \\ P_{00}&=\frac{1}{\pi a^2}\int_0^a\int_0^{2\pi}\bar{p}(r,\theta)r\,d\theta\,dr. \end{aligned} \tag{15}$$

It is assumed that the tube either extends to infinity from $z=0$, or (equivalently) is terminated in its characteristic impedance.

In the usual case confronted in practice, the pressure will be symmetrically distributed about the z axis so that (assuming propagation in the positive z direction)

$$\begin{aligned} p=\sum_{n=1}^{\infty} & P_nJ_0\left(\frac{W_nr}{C}\right)\exp\{jW[t-(z/V_n)]\} \\ & +P_0\exp\{jW[t-(z/C)]\}. \end{aligned} \tag{16}$$

[6] E. G. Churchill, *Fourier Series and Boundary Value Problems* (McGraw-Hill Book Company, Inc., New York, 1941).

[7] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1943), p. 145.

In this case the roots of $J_0'(x)$ are the roots of $J_1(x)$, and

$$J_1(W_n a/C) = J_1(2\pi a/\lambda n) = 0. \tag{17}$$

The first root of $J_1(x)$ is $x = 3.832$; if $a = 6''$ we find (using $c = 1100$ ft./sec.) $f_{01} = 1340$ cycles. It is clear that the assumption of pure plane wave propagation for high frequencies is not valid. In the light of the theory developed above we consider what happens to a sound wave traveling in a circular pipe when the radius is increased from a_1 to a_2 at $z = z_1$. For the sake of simplicity we consider that the incident wave $P^{(0)}$ consists only of a plane wave,

$$p^{(0)} = P_0^{(0)} \exp\{jW[t-(z/C)]\}, \tag{18}$$

either because of the method of excitation or because a_1 is sufficiently small and z_1 sufficiently large that all higher modes are attenuated. The extension of the analysis to the more general case is exactly analogous to this simple case. Irrespective of whether higher modes can be propagated in either section, they will be set up to satisfy the boundary conditions at the discontinuity and, even though attenuated, will determine the phase of reflected and transmitted waves. Assuming symmetry we write for the reflected wave

$$p^{(1)} = P_0^{(1)} \exp\left\{jW\left[t-\frac{|z-z_1|}{C}\right]\right\} + \sum_{n=1}^{\infty} P_n^{(1)} J_0\left(\frac{W_n^{(1)} r}{C}\right) \times \exp\left\{jW\left[t-\frac{|z-z_1|}{V_n^{(1)}}\right]\right\} \tag{19}$$

and for the transmitted wave we write

$$p^{(2)} = P_0^{(2)} \exp\left\{jW\left[t-\frac{|z-z_1|}{C}\right]\right\} + \sum_{n=1}^{\infty} P_n^{(2)} J_0\left(\frac{W_n^{(2)} r}{C}\right) \times \exp\left\{jW\left[t-\frac{|z-z_1|}{V_n^{(2)}}\right]\right\}, \tag{20}$$

where $W_n^{(1)}$ and $W_n^{(2)}$ are determined by using a_1 and a_2, respectively, in (17). The absolute value signs used with $(z-z_1)$ assure attenuation when V_{0n} is imaginary.

The boundary conditions imposed by the discontinuity are

$$p^{(0)} + p^{(1)} = p^{(2)} \quad \text{at} \quad z = z_1, \quad 0 \le r \le a_1, \tag{21}$$

$$q_z^{(2)} \sim \frac{\partial p^{(2)}}{\partial z} = 0 \quad \text{at} \quad z = z_1, \quad a_1 \le r \le a_2. \tag{22}$$

To determine the coefficients $P_n^{(1)}$ and $P_n^{(2)}$ we first multiply both sides of (21) by $J_0(W_s^{(1)} r/C) r dr$ and integrate from $r = 0$ to $r = a_1$; evaluating the integrals from reference 7, p. 145, we obtain $(s \neq 0)$.

$$\frac{P_s^{(1)}}{2}\left[a_1 J_0\left(\frac{W_s^{(1)} a_1}{C}\right)\right]^2 = Ca_1 \times \sum_{t=1}^{\infty}\left[\frac{J_0\left(\frac{W_s^{(1)} a_1}{C}\right) J_0\left(\frac{W_t^{(2)} a_1}{C}\right)}{[W_t^{(2)}]^2 - [W_s^{(1)}]^2}\right] W_t^{(2)} P_t^{(2)}. \tag{23}$$

If $s = 0$ we multiply both sides of (21) by $r dr$ and integrate from $r = 0$ to $r = a_1$ to obtain

$$[P^* + P_0^{(1)} - P_0^{(2)}]\frac{a_1^2}{2} = Ca_1 \sum_{t=1}^{\infty} J_1\left(\frac{W_t^{(2)} a_1}{C}\right)\frac{P_t^{(2)}}{W_t^{(2)}}, \tag{24}$$

where

$$P^* = P_0^{(0)} \exp(-jWz_1/C). \tag{25}$$

We next differentiate both sides of (21) with respect to z, multiply both sides of the equation by $J_0(W_t^{(2)} r/C) r dr$, and integrate from $r = 0$ to $r = a_1$. Now because of (22) we can extend the range of integration on the right-hand side of our equation to $r = a_2$, thus enabling us to take advantage of orthogonality. Evaluating the integrals as in the previous case we obtain

$$[P^* - P_0^{(1)}]\left[\frac{a_1 J_1\left(\frac{W_t^{(2)} a_1}{C}\right)}{W_t^{(2)}}\right] - a_1 C W_t^{(2)} \times \sum_{n=1}^{\infty}\left[\frac{J_0\left(\frac{W_n^{(1)} a_1}{C}\right) J_1\left(\frac{W_t^{(2)} a_1}{C}\right)}{[W_t^{(2)}]^2 - [W_n^{(1)}]^2}\right]\frac{P_n^{(1)}}{V_n^{(1)}} = \frac{P_t^{(2)}}{2V_t^{(2)}}\left[a_2 J_0\left(\frac{W_t^{(2)} a_2}{C}\right)\right]^2. \tag{26}$$

In the case $t=0$ we use rdr for a multiplier and the same ranges of integration to obtain

$$a_1^2[P^*-P_0^{(1)}]=a_2^2P_0^{(2)}. \tag{27}$$

It is interesting to note that (27) is the equation of continuity of volume flow when it is assumed that only plane wave propagation occurs; in this simple case the equation for continuity of pressure would demand

$$P^*+P_0^{(1)}=P_0^{(2)} \tag{28}$$

and (27) and (28) suffice to determine $P_0^{(1)}$ and $P_0^{(2)}$; if a_1 and a_2 are very small compared to the wave-length, (23) and (26) become identities while (24) reduces to (28).

To determine the coefficients $P_n^{(1)}$ we substitute $P_t^{(2)}$ as given by (26) in (23) to obtain

$$A_{0s}P_0^{(1)}+\sum_{n=1}^{\infty}A_{ns}P_n^{(1)}=A_{0s}P^* \qquad (s=1,\ 2,\ \cdots\infty), \tag{29}$$

while if $s=0$ we substitute $P_t^{(2)}$ from (25) and $P_0^{(2)}$ from (27) in (24) to obtain

$$A_{00}P_0^{(1)}+\sum_{n=1}^{\infty}A_{n0}P_n^{(1)}=(A_{00}-1)P^*, \tag{30}$$

where

$$A_{00}=\left\{\left(1+\frac{a_1^2}{a_2^2}\right)\frac{1}{2}+\frac{2C}{a_2^2}\sum_{t=1}^{\infty}\left[\frac{J_1\left(W_t^{(2)}\frac{a_1}{C}\right)}{W_t^{(2)}J_0\left(\frac{W_t^{(2)}a_2}{C}\right)}\right]^2V_t^{(2)}\right\}, \tag{31}$$

$$A_{n0}=\frac{2C^2}{a_2^2}\sum_{t=1}^{\infty}\left\{\left[\frac{J_1\left(\frac{W_t^{(2)}a_1}{C}\right)}{J_0\left(\frac{W_t^{(2)}a_2}{C}\right)}\right]^2\times\frac{J_0\left(\frac{W_n^{(1)}a_1}{C}\right)}{[[W_t^{(2)}]^2-[W_n^{(1)}]^2]}\frac{V_t^{(2)}}{V_n^{(1)}}\right\}, \tag{32}$$

$$A_{0s}=\frac{2C\cdot1}{a_2^2}\sum_{t=1}^{\infty}\left\{\left[\frac{J_1\left(\frac{W_t^{(2)}a_1}{C}\right)}{J_0\left(\frac{W_t^{(2)}a_2}{C}\right)}\right]^2\times\frac{V_t^{(2)}}{[[W_t^{(2)}]^2-[W_s^{(1)}]^2]}\right\}, \tag{33}$$

$$A_{ns}=\frac{2C^2\cdot1}{a_2^2}\sum_{t=1}^{\infty}\left\{\left[\frac{J_1\left(\frac{W_t^{(2)}a_1}{C}\right)}{J_0\left(\frac{W_t^{(2)}a_2}{C}\right)}\right]^2\times\frac{J_0\left(\frac{W_n^{(1)}a_1}{C}\right)}{[[W_t^{(2)}]^2-[W_s^{(1)}]^2][[W_t^{(2)}]^2-[W_r^{(1)}]^2]}\times\frac{V_t^{(2)}}{V_n^{(1)}}\right\}+\delta_s{}^n\left[\frac{1}{2}J_0\left(\frac{W_s^{(1)}a_1}{C}\right)\right]. \tag{34}$$

Equations (29) and (30) represent an infinite set of simultaneous equations which determine the coefficients $P_n^{(1)}$. Having $P_n^{(1)}$, $P_n^{(2)}$ can be obtained from (26) and (27). The infinite set of simultaneous equations must be solved by iteration. (They should first be normalized by dividing through by P_1^*.) From physical reasoning it may be inferred that the coefficients $P_n^{(1)}$ decrease monotonically with increasing n so that we can start our process by assuming that all $P_n^{(1)}$ for n greater than n_0 vanish and solving the first n_0 equations of our infinite set to obtain a first approximation to $P_0^{(1)}$, $P_1^{(1)}$, $\cdots$, $P_{n_0-1}^{(1)}$. These values are then substituted in the (n_0+1) equation to determine the first approximation to $P_{n_0}^{(1)}$; these values of $P_1-P_{n_0}$ can then be substituted in the n_0+2 equation to obtain $P_{n_0}+1$, etc. This last process of iteration can be broken off at any point and the values of $P_{n_0}^{(1)}$, $P_{n_0+1}^{(1)}$, etc., thus obtained can be substituted back in the first n_0 equations, being treated as constant terms, second approximations to P_0, P_1, $\cdots P_{n_0-1}$ can be obtained and a new iteration made. The rapidity

of convergence of this iteration process will depend on the ratios of a_1 and a_2 to the wavelength, being good if these ratios are small. It has already been assumed that a_1 is small enough so that only plane waves are propagated in the first section; if the same assumption is valid for a_2 it will probably suffice to take $n_0=2$ or 3 and use the next equation to check the accuracy of the first approximation. [Even though the series in (30)–(33) converges rather rapidly the labor involved will be tremendous if n_0 is taken very large.] The reflection coefficient in this particular case is then (for the plane wave)

$$r=\frac{p^{(1)}}{p^{(0)}}\bigg|_{z=z_1}=\frac{P_0^{(1)}}{P^*}, \tag{35}$$

while the transmission coefficient is (for the plane wave)

$$\tau=\frac{p^{(2)}}{p^{(0)}}\bigg|_{z=z_1}=\frac{P_0^{(2)}}{P^*}. \tag{36}$$

Both of these coefficients are complex and give both phase and amplitude of the reflected and transmitted waves. In the more general case where higher modes are propagated, reflection and transmission coefficients may be defined for each mode, but any attempt to define over-all coefficients will fail to describe completely the physical situation.

As a second example we calculate the effect of a thin baffle plate across a circular tube at $z=z_1$, where the plate contains a concentric hole of radius b. It is clear that this problem cannot be solved by using continuity of pressure and continuity of volume flow, or rather these conditions indicate that there is no reflection, a rather poor approximation to the truth. We assume a plane incident wave of the form of (18); the reflected and transmitted waves will then be of the form of (19) and (20), except that the cut-off frequencies and phase velocities will be the same on both sides of the discontinuity so that $p_0^{(1)}$ and $p_0^{(2)}$ differ only in their coefficients $P_n^{(1)}$ and $P_n^{(2)}$. The boundary conditions imposed by the discontinuity are

$$p^{(0)}+p^{(1)}=p^{(2)} \qquad \text{at } z=z_1,\ 0\le r\le b, \tag{37}$$

$$\frac{\partial}{\partial z}[p^{(0)}+p^{(1)}]=\frac{\partial p^{(2)}}{\partial z}=0 \quad \text{at } z=z_1,\ b\le r\le a. \tag{38}$$

It is apparent by inspection that the boundary condition imposed be differentiating both sides of (37) and combining the result with (38) can be satisfied by demanding

$$P_n^{(2)}=\delta_n{}^0P^*-P_n^{(1)} \tag{39}$$

although this equation may be easily obtained by the method of the preceding problem. To evaluate $P_n^{(1)}$ we differentiate $p^{(0)}+p^{(1)}$ with respect to z, set $z=z_1$, multiply by $J_0(W_nr/C)rdr$, and demand that the integral of the resulting quantity be the same when taken over the two ranges $r=0$ to $r=a$ and $r=0$ to $r=b$. Evaluating the integrals we obtain the set of equations

$$\frac{bJ_1\left(\frac{W_sb}{C}\right)}{W_s}P_0^{(1)}+Cb\sum_{n=1}^{\infty}\frac{\left[W_sJ_0\left(\frac{W_nb}{C}\right)J_1\left(\frac{W_sb}{C}\right)-W_nJ_0\left(\frac{W_sb}{C}\right)J_1\left(\frac{W_nb}{C}\right)\right]}{[W_s^2-W_n^2]}\frac{(1-\delta_s{}^n)}{V_n}P_n^{(1)}$$

$$+\frac{1}{2V_s}\left[b^2J_0^2\left(\frac{W_sb}{C}\right)+b^2J_1^2\left(\frac{W_nb}{C}\right)-a^2J_0^2\left(\frac{W_sa}{C}\right)\right]P_s^{(1)}=\frac{bJ_1\left(\frac{W_sb}{C}\right)}{W_s}P^* \quad (s=1,\,2,\,3,\,\cdots\infty). \tag{40}$$

In the case $s=0$ we use rdr as a multiplier and repeat the operations in the last paragraph to obtain

$$(b^2-a^2)P_0^{(1)}+2C^2b\times\sum_{n=1}^{\infty}\frac{J_1\left(\frac{W_nb}{C}\right)}{W_nV_n}p_n^{(1)}=(b^2-a^2)P^*. \tag{41}$$

Equations (40) and (41) represent an infinite set of simultaneous equations which can be solved by the procedure previously suggested.

The press of other work has kept the author from making numerical calculations or experimental verification of the preceding theory. Hartig and Swanson have made some measurements of high frequency sound fields in tubes and

verified a pressure distribution as given by (11).[3] It should also be pointed out that Lord Rayleigh[8] pointed out that a pressure distribution of the form of (11) should exist in a circular tube and gives credit to Duhamel[9] for the original idea. Nevertheless, the concepts of "transverse" modes of propagation (in addition to the usual plane wave) seem to have lain dormant until the event of increased interest in the parallel, but more complicated, electromagnetic problems in wave guides.[10]

[8] Lord Rayleigh, *Theory of Sound* (Macmillan, London, 1878), Vol. 2.
[9] J. M. C. Duhamel, Liouville J. Math. **14**, 67 (1849).
[10] See, for example, W. L. Barrow, Proc. I. R. E. **24**, 1298 (1936).

Supersonic Transmission at Oblique Incidence Through a Solid Plate in Water*

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The selective transmission of oblique supersonic waves through a solid plate immersed in water is studied theoretically and experimentally over a frequency range from 1.0 to 3.0 megacycles. Three cases are considered: (1) transmission for angles of incidence less than the critical angles for both dilatational and shear waves; (2) transmission for angles of incidence between the two critical angles; (3) transmission for angles greater than both critical angles. In the frequency region studied good agreement is found between the measured transmission, using the torsion disk method, and the theoretical values. Particular interest attaches to case (3). Study is also made of the supersonic transmission through a stratified medium consisting of a set of solid plates immersed in water. The acoustical Bragg law is investigated and verified experimentally.

INTRODUCTION

THE transmission of plane elastic waves through a plane parallel solid plate immersed in a liquid is a special case of the transmission through a stratified medium consisting of alternate plane parallel solid and liquid layers. Such a structure if infinite in extent is an elastic wave filter with pass and attenuation bands characterized by a transmission function $\cos W$ which for normal incidence has the form[1]

$$\cos W=\cos 2k_1l_1\cdot\cos 2k_2l_2 -\tfrac{1}{2}(Z_1/Z_2+Z_2/Z_1)\sin 2k_1l_1\cdot\sin 2k_2l_2, \quad (A)$$

where $2l_1$=width of solid layer, $2l_2$=width of liquid layer, $k_1=2\pi\nu/c_1$, $k_2=2\pi\nu/c_2$, with c_1 and c_2 the compressional wave velocities in the two media, respectively, ν the frequency, and $Z=\rho c/S$, the acoustic plane wave impedance for a wave front area S. The pass bands correspond to the frequency regions for which $|\cos W|\leq 1$, whereas the attentuation bands are characterized by $|\cos W|>1$. Examination of (A) discloses that the pass bands occur in the neighborhood of the frequencies for which $\sin 2k_1l_1=0$ and $\sin 2k_2l_2=0$, i.e., the characteristic frequencies of each layer in the structure. The band widths depend on the factor $\frac{1}{2}(Z_1/Z_2+Z_2/Z_1)$. Thus if this is unity ($Z_1=Z_2$) there is only one band, i.e., a transmission band of infinite width. The greater the difference between Z_1 and Z_2, the narrower are the pass bands and the wider the intervening attenuation bands. In every case, of course, the structure is a *low pass* filter in the sense that the band beginning with zero frequency is a pass band.

If the stratified medium is a finite structure imbedded in an infinite medium the transmission ratio[2] P_r when computed and plotted as a function of frequency proves to be very small in the attenuation bands of the corresponding infinite structure; in the pass bands, P_r is much larger on the average and possesses a structure characterized by a number of peaks equal to the number of sections, where each section consists of a pair of layers of the two materials making up the stratified medium.

A low frequency filter of the type just discussed was recently constructed at Brown University in the form of five identical sections each consisting of a right circular cylinder of birchwood 1.27 cm in diameter and 30 cm long coupled to a cylinder of aluminum of the same dimensions. The $\cos W$ curve for the equivalent infinite structure indicates a pass band extending from zero to 2500 cycles. The transmission in this band was studied by suspending the structure vertically with the driving mechanism at the top and the bottom left free. Piezoelectric phonograph pick-ups were used as detectors to measure the transmission. The experimental cut-off was found to agree extremely well with

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[1] Cf. Lindsay, Lewis, and Albright, J. Acous. Soc. Am. 5, 202 (1934).

[2] Cf. R. B. Lindsay, J. App. Phys. 9, 612 (1938).

the theoretically predicted one. Four well-marked peaks were found in the transmission region at frequencies of approximately 300, 900, 1400, and 1900 cycles, respectively, in moderately good agreement with the theoretical peaks. The fifth theoretical peak at 2400 cycles was not observed.

Theoretical studies of transmission of plane elastic waves at oblique incidence through a stratified medium have previously been carried out by one of the authors[3] with the general result that the structure behaves as a low pass, band pass filter with pass and attenuation bands depending in characteristic fashion on the angle of obliquity.

THEORY

The present paper is intended as a report on the experimental verification of the theory mentioned in the introduction, treating transmission through a single plate in water as a special case of a solid-liquid stratified medium. It will be necessary to present here only a brief review of the relevant transmission formulas.[4] If the normal components of the particle velocities of the incident and transmitted waves in the water on the two sides of the plate at the surface are denoted by $\dot{\zeta}_0$ and $\dot{\zeta}_2$, respectively, the power transmission ratio is

$$P_r = |\dot{\zeta}_2|^2/|\dot{\zeta}_0|^2. \tag{1}$$

By the use of the method outlined in reference 3, the following general formula is obtained:

$$\frac{\dot{\zeta}_2}{\dot{\zeta}_0} = \frac{1}{\alpha + \frac{i}{2}(\beta/\cos\theta_1 + \gamma\cos\theta_1)}, \tag{2}$$

where θ_1 is the angle of incidence from the water to the plate and

$$\alpha = \frac{M_1 e^{iP} + M_2 e^{-iP}}{M_1 + M_2}, \tag{3}$$

$$\beta = \frac{i(M_1^2 e^{iP} - M_2^2 e^{-iP})}{Z(M_1 + M_2)}, \tag{4}$$

$$\gamma = -\frac{2Z\sin P}{M_1 + M_2}. \tag{5}$$

In these expressions and those which follow

$$P = k_\delta l\cos\theta_\delta, \quad Q = k_s l\cos\theta_s, \tag{6}$$

where θ_δ is the angle of refraction for the dilatational wave in the solid and θ_s is the corresponding refractive angle for the shear wave. The thickness of the plate is l and $k_\delta = 2\pi\nu/c_\delta$, while $k_s = 2\pi\nu/c_s$, and c_δ and c_s are the dilatational and shear wave velocities in the plate, respectively. Also $Z = \rho_1 c_1(c_\delta\cos\theta_\delta + c_s\cdot\sin 2\theta_\delta\cdot\sin\theta_s/\cos 2\theta_s)$, where ρ_1 is the density of the water and c_1 the elastic wave velocity therein. Finally,

$$M_1 = -\rho_s c_\delta^2\cos 2\theta_s - \frac{i\mu\sin 2\theta_\delta\cdot\sin 2\theta_s}{\cos 2\theta_s\cdot\sin Q}e^{-iP} + \frac{i\mu\sin 2\theta_\delta\cdot\sin 2\theta_s}{\cos 2\theta_s\cdot\sin Q}\cdot\cos Q, \tag{7}$$

$$M_2 = -\rho_s c_\delta^2\cos 2\theta_s + \frac{i\mu\sin 2\theta_\delta\cdot\sin 2\theta_s}{\cos 2\theta_s\cdot\sin Q}e^{iP} - \frac{i\mu\sin 2\theta_\delta\cdot\sin 2\theta_s}{\cos 2\theta_s\cdot\sin Q}\cos Q. \tag{8}$$

In Eqs. (7) and (8) ρ_s is the density and μ the shear modulus of the solid plate. It is interesting to note that α, β, and γ satisfy the following relation:

$$\alpha^2 + \beta\gamma = 1. \tag{9}$$

In the evaluation of P_r from the ratio given in (2) it is convenient to consider separately the three following cases:

(A) The angle θ_1 less than the critical angles for both dilational and shear waves, so that θ_s and θ_δ are both real.

(B) The angle θ_1 greater than the critical angle for the dilatational wave but less than the critical angle for the shear wave, i.e., θ_s real and θ_δ complex.

(C) The angle θ_1 greater than both critical angles, i.e., θ_s and θ_δ both complex.

[3] R. B. Lindsay, J. Acous. Soc. Am. 11, 178 (1939).

[4] It should be mentioned that theoretical results similar to those of the present section have already been obtained by H. Reissner [Helv. Phys. Acta. 11, 140 (1938)].

The relevant P_r formulas follow:

$$\text{(A)}\quad P_r=\frac{(a_0\sin Q+b_0\sin P)^2}{(a_0\cos P\sin Q+b_0\cos Q\cdot\sin P)^2+\left[\frac{f_0}{2Z}\cdot\sin P\cdot\sin Q+\frac{a_0b_0}{Z}(1-\cos Q\cdot\cos P)\right]^2}, \tag{10}$$

where

$$a_0=\rho_s c_\delta^2\cos 2\theta_s,\quad b_0=\mu\sin 2\theta_\delta\cdot\tan 2\theta_s,\quad f_0=a^2+b^2+Z^2. \tag{11}$$

$$\text{(B)}\quad P_r=\frac{(a_1\sin Q-b_1\sinh P_1)^2}{(a_1\sin Q\cdot\cosh P_1-b_1\cos Q\cdot\sinh P_1)^2+\left[\frac{f_1}{2Z'}\sinh P_1\sin Q+\frac{a_1b_1}{Z'}(1-\cosh P_1\cos Q)\right]^2}, \tag{12}$$

where

$$a_1=\rho_s c_\delta^2\cos 2\theta_s,\quad b_1=\mu\sinh 2y\cdot\tan 2\theta_s,\quad f_1=a_1^2-b_1^2-Z'^2. \tag{13}$$

$$Z'=\rho_1 c_1\left(c_\delta\sinh y+c_s\cdot\frac{\sinh 2y\sin\theta_s}{\cos 2\theta_s}\right),\quad P_1=k_\delta l\sinh y,\quad \cos\theta_\delta=-i\sinh y. \tag{14}$$

$$\text{(C)}\quad P_r=\frac{(a_2\sinh Q_1-b_2\sinh P_1)^2}{(a_2\sinh Q_1\cosh P_1-b_2\cosh Q_1\cdot\sinh P_1)^2+\left[\frac{f_2}{2Z''}\sinh P_1\sinh Q_1-\frac{a_2b_2}{Z''}(\cosh P_1\cosh Q_1-1)\right]^2}, \tag{15}$$

where

$$a_2=\rho_s c_\delta^2\cosh 2v,\quad b_2=\frac{\mu\sinh 2y\cdot\sinh 2v}{\cosh 2v}. \tag{16}$$

$$Z''=\rho_1 c_1\left(c_s\cdot\frac{\sinh 2y\cdot\cosh v}{\cosh 2v}-c_\delta\sinh y\right),\quad f_2=a_2^2+b_2^2-Z''^2, \tag{17}$$

$$P_1=k_\delta l\sinh y,\quad Q_1=k_s l\sinh v,\quad \cos\theta_s=-i\sinh v.$$

An interesting result of the analysis is that transmission results for certain frequency ranges in *all* three cases, even in case C. Comparison between theoretical and experimental values is given in detail below.

FIG. 1. Experimental tank.

EXPERIMENTAL DETAILS

The source of the high frequency radiation used in the experimental measurements consisted of a set of piezoelectric quartz crystals with resonance frequencies appropriately distributed over the frequency range from about 0.7 megacycle to 3.0 megacycles. A special type crystal holder was constructed to facilitate rapid interchange of crystals. The crystals were driven by a Dow electron-coupled oscillator possessing moderately good frequency stability. After the initial warming-up period of about two hours, the frequency variation was less than 1/10 percent.

The radiation was produced in a tank, 60 cm×37 cm×15 cm, with glass walls and filled with water. The source was placed at one end of the tank and a specially constructed sound trap was placed at the other end to prevent

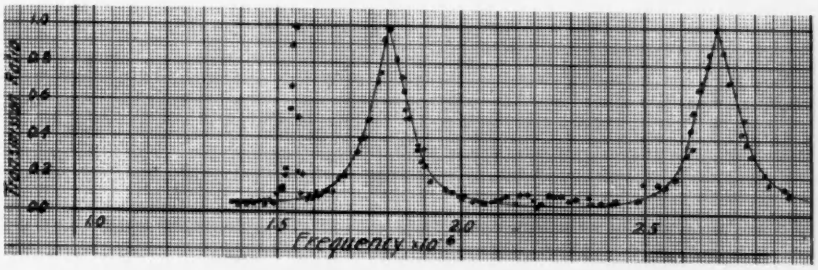

FIG. 2. Transmission through a single plate (glass). Theory ——; Experiment o o o. $\theta_1=0°$, $l=0.333$ cm.

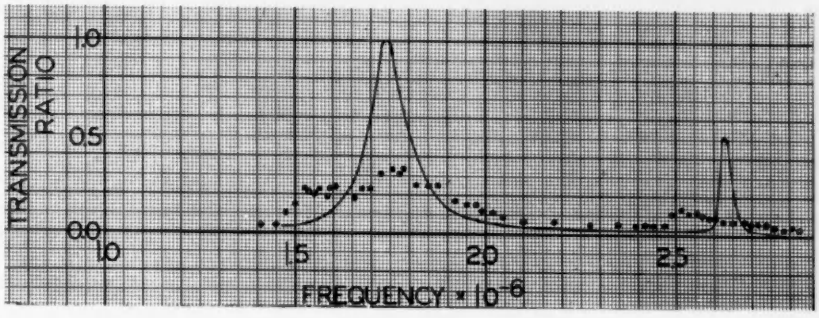

FIG. 3. Transmission through a single plate of glass. Region A. Theory ——; Experiment o o o. $\theta_1=12.2°$, $\theta_a=29.1°$, $\theta_\delta=57.4°$, $l=0.333$ cm.

the formation of a standing wave pattern. The trap consisted of a collection of glass plates oriented in such a way as to reflect the sound out of the incident beam and direct it into one corner of the tank.

At the lowest frequency used the angular spread of the supersonic beam did not exceed 7°. At the highest frequency employed the corresponding figure was 1.5°. This rendered possible the use of a rather shallow tank without incurring the risk of material reflection at the side walls and bottom.

The radiation was detected by means of a torsion disk constructed in conventional fashion from two thin mica sheets fastened to opposite sides of a thin aluminum ring, 0.476 cm in radius. The suspension was a fine gold wire, 13.3 cm long, suspended from a graduated torsion head and counterpoised at the other end with a small brass cylinder. The calibration of the disk follows the formula

$$I=(1.26\times10^{-6})\theta \text{ watt/cm}^2, \qquad (18)$$

giving the intensity of the beam as a function of the angle θ (expressed in degrees) through which the torsion head had to be rotated to bring back the disk into a position normal to the sound field (i.e., beam direction). This detector proved competent to measure relative intensities to an accuracy of 1 percent.

A photograph of the experimental tank and equipment is shown in Fig. 1. This exhibits the crystal holder, the glass plate, the torsion disk, and the sound trap. The frequency of the oscillator was measured by an absorption type wave meter. The experimental P_r values were obtained by taking the ratio of the I values [from (18)] for the glass plate in the beam and out of the beam, respectively.

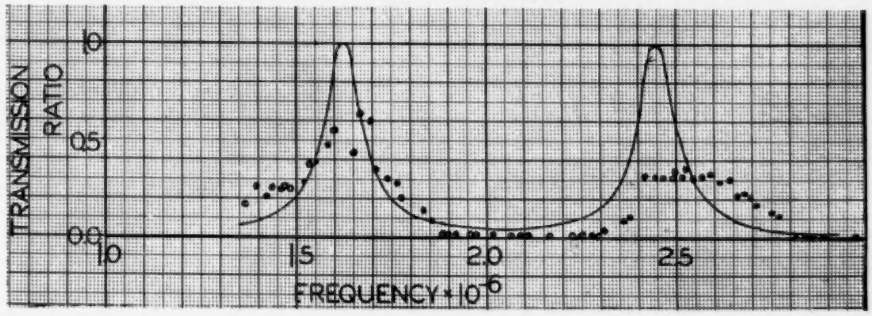

FIG. 4. Transmission through a single plate of glass. Region B. Theory ——; Experiment o o o. $\theta_1=19.8°$, $\theta_a=51.4°$, θ_δ =complex, $l=0.333$ cm.

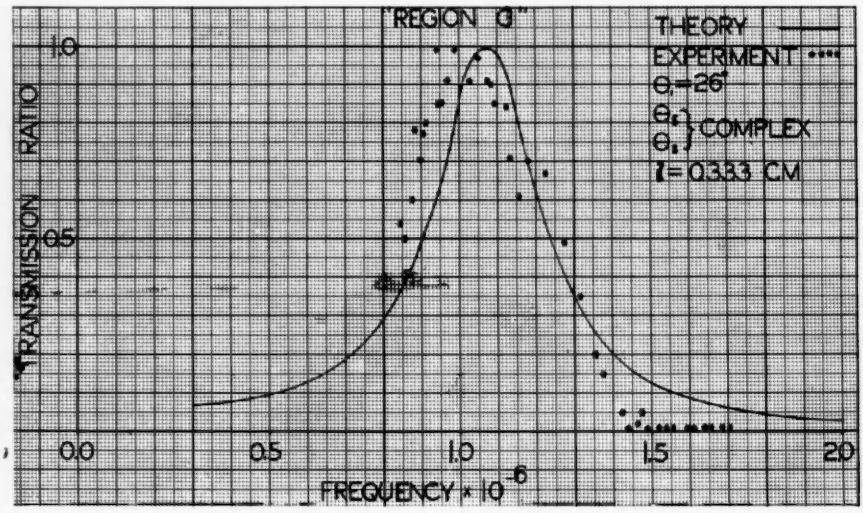

FIG. 5. Transmission through a single plate of glass.

RESULTS AND COMPARISON OF THEORY AND EXPERIMENT

To evaluate the theoretical values of P_r it is necessary to know the appropriate elastic constants and elastic wave velocities. The dilatational velocity (c_δ) in the glass was calculated from the measured frequency for maximum transmission at normal incidence using the formula

$$\lambda_\delta = \frac{2l}{n} = c_\delta/\nu, \tag{19}$$

where n is integral. The accuracy with which this can be done is shown in Fig. 2, where the full curve gives the theoretical transmission for the case of normal incidence and the circles represent the measured values of P_r. The corresponding value of c_δ proves to be 5.98×10^5 cm/sec. The experimental peak at 1.55 megacycles does not appear to correspond to any theoretical peak and is probably due to a certain lack of homogeneity in the glass. The constants μ and c_s were chosen so as to provide the best fit between experimental and theoretical P_r values for case B. The values finally chosen were $\mu=2.98\times10^{11}$ dynes/cm^2 and $c_s=3.45\times10^5$ cm/sec. The order of magnitude of these agrees with the range of values quoted in the *International Critical Tables*.

The nature of the agreement between theory and experiment is exhibited in the three following figures (Figs. 3–5) for the three cases A, B, and C, respectively. The experimental maxima follow the theoretically predicted ones in reasonably satisfactory fashion. Particularly interesting is the transmission in case C where both θ_s and θ_δ are complex. Here the theory predicts a single transmission peak, which is very well substantiated by experiment. This suggests that for thick plates such as were used in the present investigation, the theory of Sanders[5] which attributes transmission in this angular region to flexural vibrations of the plate as a whole, does not apply or at any rate is not particularly significant.

TRANSMISSION THROUGH A SET OF GLASS PLATES IN WATER

It may be of interest to include in the present paper a brief discussion of attempts to study supersonic transmission through a set of plane parallel glass plates immersed in water, i.e., a stratified medium of the kind mentioned in the introduction. We encounter at once a certain difficulty in the experimental testing of the theory set forth in reference 3. This results from the fact that in the frequency range from 1 to 3 megacycles water is a strong absorber of supersonic radiation. Now the action of a stratified medium filter depends essentially on the reinforcement of multiply-reflected waves in the component media: In other words, the transmission is primarily a *reactance* and not a *dissipative*

[5] F. H. Sanders, Can. J. Research **17**, 179 (1939).

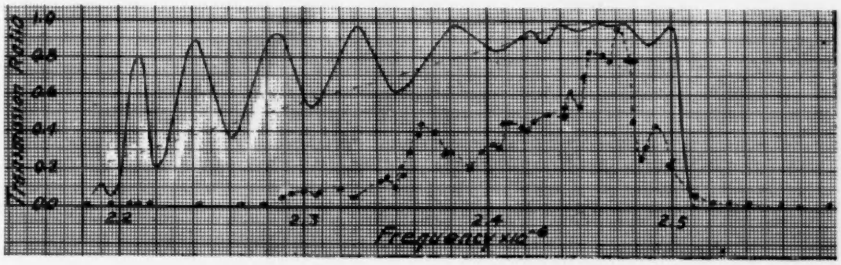

FIG. 6. Transmission through a glass-water filter (10 sections). Theory ——; Experiment o o o. $\theta_1=0°$, $2l_1=0.0246$ cm, $2l_2=0.118$ cm.

action. The presence of purely dissipative attenuation-like absorption will materially decrease P_r in the transmission bands which result from the multiple reflections in the absorbing medium, in the present case the water. This presumption has been well borne out by experimental observations on a glass-water structure consisting of 10 plane parallel glass plates 0.025 cm in thickness separated by water layers 0.12 cm in thickness. No transmission was found in the theoretically computed transmission bands corresponding to the water layers. On the other hand, typical filtration action was found in the transmission bands corresponding to the glass plates and in the usual attenuation bands. Figure 6 shows the theoretical and experimental P_r values for the transmission band from 2.2 to 2.5 megacycles. The experimental cut-off is fairly sharp and the experimental points show up some of the theoretically predicted structure.

TABLE I.

ν (megacycles)	λ (water)	θ (calc.)	θ (obs.)
2.7	0.0553 cm	30°13′	29°20′
2.6	0.0577 cm	25°37′	26°48′
2.5	0.0600 cm	20°22′	20°42′
2.4	0.0635 cm	12°26′	11°30′

THE ACOUSTICAL BRAGG LAW

In reference 3 the theory of the transmission of oblique elastic waves in stratified media was applied to the special case of the selective reflection from a set of thin solid plates immersed in a fluid medium in which the fluid layers are considerably thicker than the plates. It was there found that selective reflection takes place for the angles of incidence given by

$$\cos\theta_1=n\lambda/2l, \tag{20}$$

where n is integral and l is the thickness of the fluid layers. This is the elastic wave analog of the Bragg law for the selective reflection of x-rays. Experimental investigation of this was undertaken by constructing a structure of eight sheets of mica of average thickness 0.0064 cm equally spaced 0.056 cm apart in water. Table I gives the calculated and observed values of the angle of maximum reflection for $n=2$. The agreement may be considered reasonably satisfactory.

The authors desire to make grateful acknowledgment of assistance rendered by Rachel T. Lindsay in the checking of the calculations and by Eugene Hsu in the preparation of certain of the drawings.

A Review of Supersonic Methods for Measuring Elastic and Dissipative Properties of Solids*

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(Received June 15, 1944)

I. INTRODUCTION

NUMEROUS dynamical methods for measuring the elasticity and damping of solids have been devised. These usually involve the excitation of longitudinal or torsional oscillations in the specimen, a bar a few cm in length, a few mm in diameter.

The fundamental frequency f of longitudinal vibration of a free bar is determined by the equation

$$E=4f^2L^2\rho(1+\tfrac{1}{2}\pi^2\sigma^2r^2/L^2), \qquad (1)$$

where E is Young's modulus, L the length of the bar, r its radius, ρ its density, and σ is Poisson's ratio. For torsional vibrations the equation is simply

$$G=4f^2L^2\rho \qquad (2)$$

with G the torsion modulus of elasticity.

Hence a measurement of the resonant frequency yields data from which the appropriate elastic modulus may be computed. If the system is one in which a steady state of vibrations is produced by a periodic external force, the resonance frequency is that at which the velocity is in phase with the force; at this frequency the amplitude is a maximum. At frequencies above and below this, the amplitude decreases; the frequency difference between the two half-power points on either side of resonance is simply related to the energy loss in the system, $\Delta E/E=2\pi(\Delta f/f)$. Thus the width of the resonance curve yields the dissipative constants of the specimen.

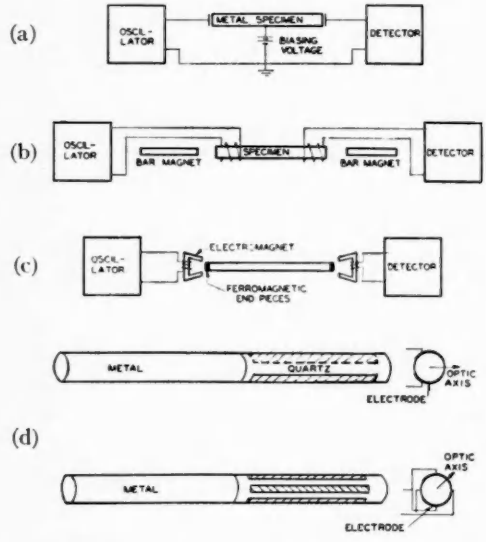

FIG. 1. Dynamical methods for measuring elastic moduli and internal damping.

II. METHODS

Several of the commonly used dynamical methods are shown in Fig. 1. The electrostatic method[1] is shown at (a). The oscillator voltage is applied to a condenser, one plate of which is the specimen, the other plate being fixed, very closely adjacent to the specimen. The alternating electrostatic force of the oscillator frequency between the condenser plates produces mechanical vibration of the specimen. Detection of the amplitude of motion is achieved by a condenser microphone, high gain amplifier combination.

A method due to Zener[2] makes use of eddy currents, shown at (b). These are excited in the conducting specimen by the coil surrounding it and connected to the oscillator. The permanent magnet produces a force on these currents, and hence on the sample. The detector operates on the eddy currents induced in the right-hand end of the bar due to its motion in the magnetic field.

An electrodynamic method[3] is indicated at (c). Its operation is similar to that of an electromagnetic loudspeaker and microphone. These three methods are suitable for producing longi-

* Presented at the twenty-ninth meeting of the Acoustical Society of America, May 13, 1944, New York, New York.

[1] R. B. Jacobs and D. Bancroft, Rev. Sci. Inst. **9**, 279 (1938).
[2] C. Zener, F. C. Rose, and R. H. Randall, Phys. Rev. **56**, 343 (1939).
[3] R. L. Wegel and H. Walther, Physics **6**, 141 (1935).

tudinal vibrations, and the first and third with suitable modifications can be used for torsional vibrations.

A method[4] utilizing the piezoelectric properties of quartz is indicated at (d), and in greater detail in Fig. 2. Here the sample is cemented to an appropriately cut cylinder of crystalline quartz. The upper arrangement uses an X-cut quartz bar and produces longitudinal vibrations; the lower uses a Y-cut bar producing torsional vibrations.

The composite oscillator, consisting of quartz and specimen affixed to it, forms part of the electric circuit of Fig. 2. The output of a constant amplitude, variable frequency oscillator is applied to the quartz in series with a non-reactive resistor. The current to the quartz is given by

$$I = C\dot{E} - k\epsilon\dot{\bar{S}}, \tag{3}$$

where C is the electrostatic capacity of the quartz, E the voltage applied to it, k a geometrical constant, ϵ a piezoelectric constant, and $\bar{S}$ the average strain along the length of the quartz bar. This current is measured with the vacuum-tube voltmeter across the series resistor, and yields data on the amplitude of mechanical vibration. The frequency of the oscillator may be determined, to a few parts per million, by comparison in the detector and a-f amplifier, with the spectrum of frequencies furnished by the quartz controlled 100-kc oscillator driving two multivibrators in series. In this way it is possible to plot quartz-current *vs.* frequency curves from which the resonant frequency, and the resonance curve breadth, for the composite oscillator may be determined. A previous knowledge of the properties of the quartz bar alone then permits the computation of the properties of the specimen alone. Thus the resonance frequency f of the specimen is determined from

$$m\frac{\tan\dfrac{\pi f_0}{f}}{\dfrac{\pi f_0}{f}} + m_Q\frac{\tan\dfrac{\pi f_0}{f_Q}}{\dfrac{\pi f_0}{f_Q}} = 0, \tag{4}$$

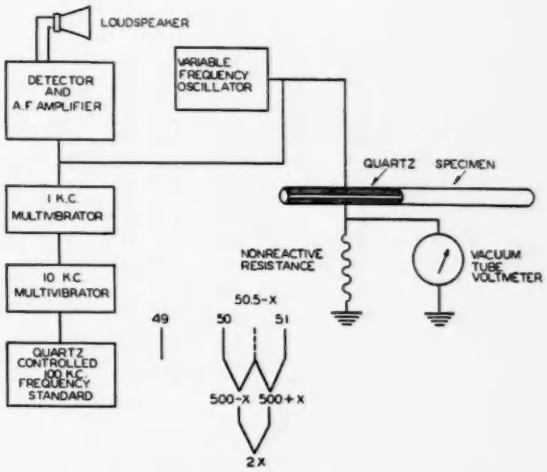

FIG. 2. Schematic diagram of circuit used with composite piezoelectric oscillator.

where f_0 is the observed frequency of the composite bar, m the mass of the specimen, and the quantities with subscript Q refer to the quartz bar alone. A similar expression exists for the decrement.

The accuracy obtainable with this method is remarkably high. It is possible to measure changes in elastic moduli to about 0.01 percent, and to measure absolute values to 0.1 percent. Decrements of the order of 10^{-4} can be measured to 1 percent.

III. TYPICAL PROBLEMS

A variety of problems in fields related to magnetism, plasticity, metallurgy, thermal properties of solids, etc., has been studied by these methods. As illustrations, a few of these investigations will be described.

In ferromagnetic materials there is an interplay between the magnetic and elastic properties, arising through magnetostriction. The composite piezoelectric oscillator has been used to investigate the variation of the elastic and damping properties of single and polycrystalline samples of nickel with the intensity of magnetization and the temperature.[5]

The results for Young's modulus are shown in Fig. 3. The various curves are for inductions which are, respectively, 0, 0.2, 0.4, 0.6, 0.8, and 1.0 times the saturation induction at the tem-

[4] L. Balamuth, Phys. Rev. **45**, 715 (1934); F. C. Rose, Phys. Rev. **49**, 50 (1936).

[5] S. Siegel and S. L. Quimby, Phys. Rev. **49**, 663 (1936).

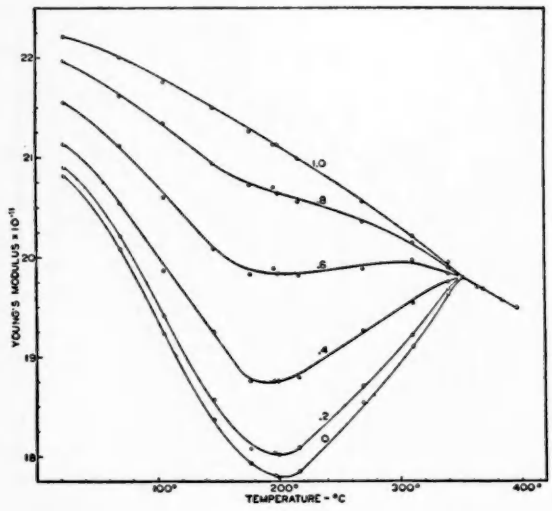

FIG. 3. Variation with temperature of Young's modulus for nickel, in various states of magnetization.

perature of measurement. The top curve is for the saturated state. A similar family of curves for the damping is shown in Fig. 4, in this case the lowest curve being for the saturated state. Evidently the high damping is almost entirely of ferromagnetic origin, and is reduced by a factor of 10 to 100 where the material is saturated or is above the Curie temperature. These effects arise through the microscopic changes in magnetization which ensue as a result of the oscillating stresses in the sample, and these results lend support to the general domain theory of magnetization.

In single crystals having a cubic lattice, the elastic moduli are not isotropic. Three principal moduli S_{11}, S_{12}, S_{44} exist, from which Young's and the torsion modulus in any direction may be computed.

$$1/E = S_{11} - 2S\Gamma, \tag{5}$$

$$1/G = S_{44} + 4S\Gamma - \frac{2S^2(\Gamma - 4\Gamma^2 + 3x)}{S_{11} - 2S\Gamma}, \tag{6}$$

where

$$\Gamma = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2,$$

$$x = \alpha^2\beta^2\gamma^2,$$

$$S = S_{11} - S_{12} - \tfrac{1}{2}S_{44},$$

α, β, γ being the direction cosines of the specimen cylinder axis with respect to the cubic axis of

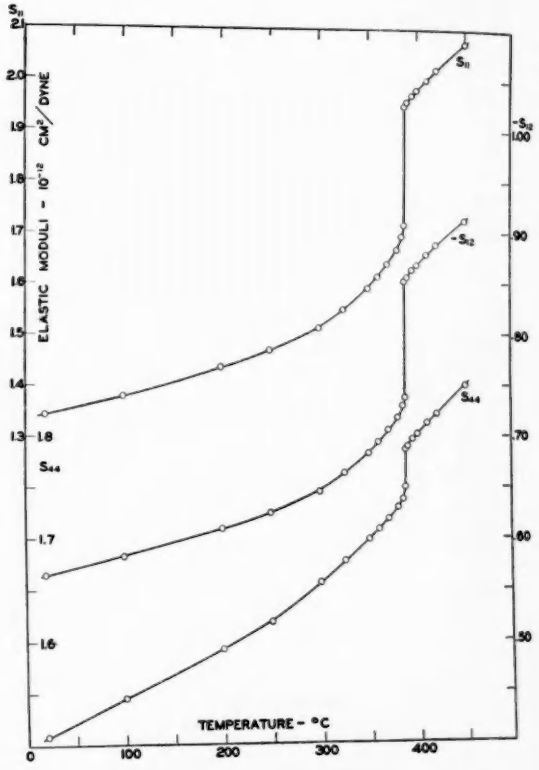

FIG. 5. Variation of the principal elastic moduli of Cu_3Au with temperature.

the lattice. These equations are applicable in the dynamical method only, and their region of validity has been discussed by Brown.[6]

These three elastic moduli for single crystals of the copper gold alloy Cu_3Au are plotted against temperature in Fig. 5.[7] This alloy, when slowly cooled, exists as an ordered cubic struc-

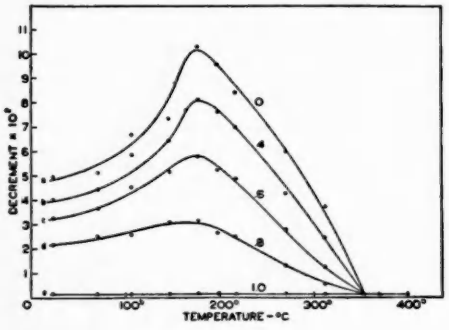

FIG. 4. Variation of damping with temperature and magnetization, for polycrystalline nickel.

[6] W. F. Brown, Phys. Rev. **58**, 998 (1940).
[7] S. Siegel, Phys. Rev. **57**, 537 (1940).

ture. As the temperature is raised it becomes increasingly disordered, and above 390°C becomes a random solid solution. From the nature of the variation of the elastic constants of the alloy with the degree of order, it is possible to check theoretical ideas on the nature of the lattice forces in the solid.

In the neighborhood of the transformation temperature the elastic moduli vary with time, at constant temperature. In this region the rates of approach to equilibrium states of order are slow enough that the kinetics of the transformation may be studied by following the variation of elastic modulus with time.[8]

From families of isothermal reaction curves obtained in this way it is possible to obtain information on the details of the transformation process. Similar experiments have been carried out during the precipitation hardening of aluminum-copper alloys, and during the recrystallization of cold-worked aluminum.

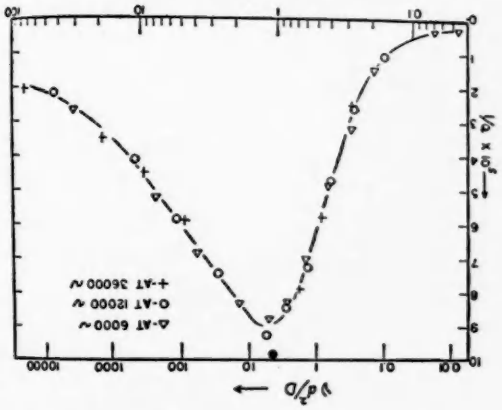

FIG. 6. The damping constant $1/Q$ for 70-30 brass samples of differing grain size, plotted against the parameter (fd^2).

The eddy current method described in reference 2 was used to study the variation of internal friction of polycrystalline brass with grain size d and with frequency f. The internal friction is plotted against the quantity (fd^2) in Fig. 6. These experiments revealed that the major portion of the internal friction of polycrystalline specimens in which the oscillating stresses are nearly homogeneous macroscopically is due to microscopic thermal currents arising from elastic anisotropy. The theory of such microscopic thermal currents had been previously developed by Zener.

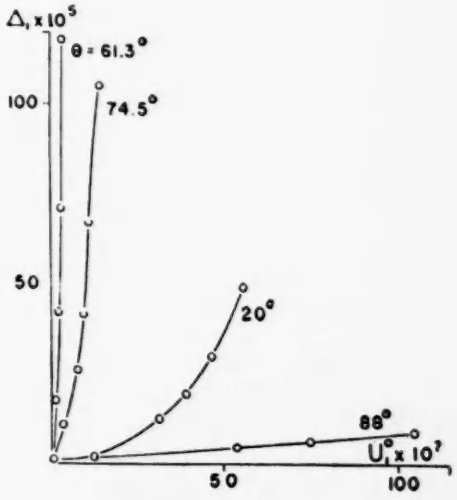

FIG. 7. Variation of log decrement with amplitude of oscillation, in zinc crystals whose cylinder axes make various angles θ with the hexagonal axis.

The internal friction in single crystals of various metals has been studied extensively by Read.[9] In such specimens the damping arises from plasticity at extremely small amplitudes, associated with the motion of imperfections in the lattice. The results for single crystals of zinc whose cylinder axis make various angles with the basal plane, upon which slip occurs, are shown in Fig. 7. The logarithmic decrement is plotted against the strain amplitude. Since the maximum resolved shear stress in the sample undergoing

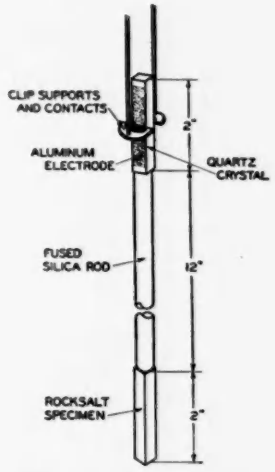

FIG. 8. Three-part piezoelectric oscillator.

[8] S. Siegel, J. Chem. Phys. **8**, 860 (1940).

[9] T. A. Read, Phys. Rev. **58**, 371 (1940).

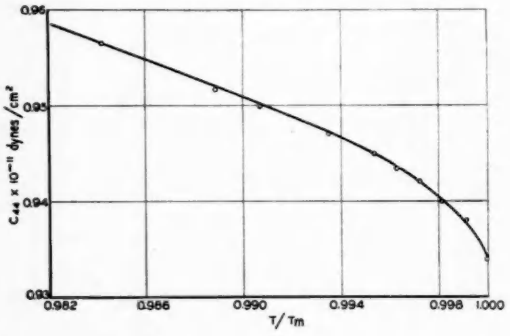

FIG. 9. Variation of C_{44} near the melting point. The abscissae show the temperature, in fractions of the melting temperature.

longitudinal oscillations lies in a plane at 45° to the cylinder axis, those crystals in which the slip plane lies closest to this direction exhibit the most marked dependence of internal friction upon amplitude.

A final example, which could only have been carried out by a dynamical method, is concerned with the elastic moduli of NaCl in the immediate neighborhood of the melting point.[10]

Older experiments, considerably below the melting point, had indicated that the shear modulus of a solid might approach zero continuously as the melting point was approached. Born and Furth[11] have developed a theory dealing with this problem, which indicates that the shear modulus may have a non-zero value just below the melting point.

The composite oscillator in this case consisted of three parts: a crystal quartz driver, an intermediate long bar of fused silica, and the rock-salt specimen; it is shown in Fig. 8. The NaCl specimen was situated at the center of a tube furnace, in a uniform temperature region; the 12″ silica bar extended into the cooler end of the furnace, where the quartz was situated. This arrangement was used because of the $\alpha \rightarrow \beta$ transformation of quartz at 570°, which prevents its use at temperatures near 904°C, the melting point of NaCl. All three moduli were measured, the torsion modulus in the (100) direction, C_{44}, in the close neighborhood of the melting point, being plotted in Fig. 9. It is evident that the shear modulus does not approach zero, but has a definite value within a fraction of a degree of the melting point. The general shape of the curve is in qualitative agreement with those calculated by Born.

These few examples may serve to indicate the versatility of the dynamical methods for investigating elastic and dissipative properties of small samples, in unusual and difficult environments, and to show how the information so obtained is of value in advancing our general knowledge of the solid state.

[10] L. P. Hunter and S. Siegel, Phys. Rev. **61**, 84 (1942).
[11] M. Born, J. Chem. Phys. **7**, 591 (1939); R. Furth, Proc. Camb. Phil. Soc. **37**, 34 (1941).

Propagation of Sound in Lined Ducts*

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(Received June 13, 1944)

INTRODUCTION

THE exact solution to the problem of the propagation of sound inside of ducts lined with sound absorbing material has been given by P. M. Morse.[1] However, the application of Morse's results is quite laborious in many practical cases and it now seems useful to develop approximate methods for the problem. For this purpose the theory of L. J. Sivian[2] is taken as the starting point. This theory is valid for frequencies such that the corresponding half-wave-lengths are larger than the longest dimension of the duct cross section. This condition is sufficient to guarantee that the pressure is approximately constant in the plane of a cross section taken normal to the duct axis. When applicable the formulas and graphs given in this paper enable one to calculate rapidly the attenuation constants for a given system. The equations of the Sivian theory were previously derived by means of analogies with the long electric transmission line. In the present paper an independent mechanical derivation is given and it is shown how to apply the theory to the case of a rectangular duct lined on each side with a different material. The variables of the problem are reduced to four dimensionless parameters and the solution of many attenuation problems can be found by reference to the curves or formulas given. Various special cases corresponding to limiting values of lining impedance are discussed. The variation of attenuation as a function of duct size is discussed and it is shown that there are two limiting cases. The first occurs when the duct is relatively unabsorbing. Here it is found that the attenuation is proportional to (L/A). The second limiting case is when the duct is highly absorbing. Here the attenuation is proportional to $(L/A)^{\frac{1}{3}}$. For the intermediate region the attenuation is a more complicated function involving the frequency and lining impedance as well as (L/A).

A method for calculating lining impedance from attenuation measurements is given.

The point of view taken in this paper is that the material presented represents mathematical deductions based on the Sivian theory and no attempt is made to compare theory and measurement. A review of the field is now being made and when this is completed the above-mentioned comparisons will be presented.

DERIVATION OF THE DIFFERENTIAL EQUATIONS

Consider a duct lined on all sides with the same sound absorbing material, having a constant cross section, which is not necessarily rectangular. In the small volume bounded by the cross-sectional planes at (x) and $(x+\Delta x)$ the pressure normal to the duct walls is assumed to be constant. (See Fig. 1.) The pressure is also assumed to be constant across any cross-sectional plane perpendicular to the duct axis. On the basis of these assumptions the following equations can be set up:

Force equation:

$$-\frac{\partial p}{\partial x}=\rho\frac{\partial^2\epsilon}{\partial t^2}. \tag{1}$$

Adiabatic equation:

$$p=-P_0\gamma\frac{\Delta V}{V}=-\rho c^2\frac{\Delta V}{V} \tag{2}$$

(where V = volume of element chosen $= A\cdot\Delta x$

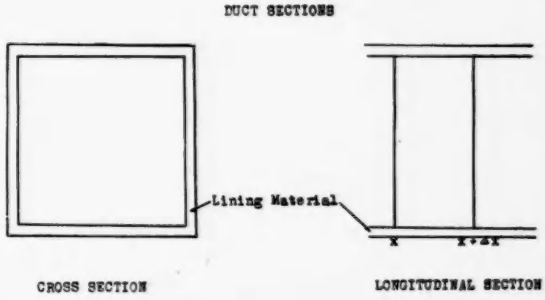

FIG. 1.

* Paper presented at the twenty-ninth meeting of the Acoustical Society of America, May 12–13, 1944, New York, New York.

† Physicist, Material Laboratory, Navy Yard, New York, New York.

[1] P. M. Morse, J. Acous. Soc. Am. **11**, 205 (1939).

[2] L. J. Sivian, J. Acous. Soc. Am. **9**, 135 (1937).

and ΔV = increment of this volume due to longitudinal and transverse displacements).

Continuity equation:

$$\Delta V=\left[\frac{\partial \epsilon}{\partial x}+\frac{L}{A}\epsilon_n\right]A\cdot\Delta x. \tag{3}$$

Impedance equation:

$$\frac{\partial \epsilon_n}{\partial t}=\frac{p}{\rho c z}. \tag{4}$$

Combining Eqs. (1)–(4) the following differential equation is obtained:

$$\frac{\partial^2 p}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}-\frac{L}{Acz}\cdot\frac{\partial p}{\partial t}=0. \tag{5}$$

For the case of a rectangular duct lined on each side with a different sound absorbing material it is apparent that the force equation and the adiabatic expansion law remain unchanged. However, a new equation of continuity and new impedance equations are required.

Continuity equation:

$$\Delta V=\left[\frac{\partial \epsilon}{\partial x}+\frac{1}{A}\{l_1(\epsilon_1+\epsilon_3)+l_2(\epsilon_2+\epsilon_4)\}\right]A\cdot\Delta x. \tag{6}$$

Impedance equations:

$$\frac{\partial \epsilon_i}{\partial t}=\frac{p}{\rho c z_i};\quad i=1,\ 2,\ 3,\ 4. \tag{7}$$

Combining (6) and (7) with (1) and (2) a differential equation identical in form with (5) is obtained, the only difference being that (z) is replaced by (z_e), where (z_e) is called the equivalent impedance and is defined by:

$$\frac{1}{z_e}=\frac{1}{\left(\frac{L}{l_1}z_1\right)}+\frac{1}{\left(\frac{L}{l_2}z_2\right)}+\frac{1}{\left(\frac{L}{l_1}z_3\right)}+\frac{1}{\left(\frac{L}{l_2}z_4\right)}. \tag{8}$$

Hence in order to make calculations for a duct differently lined on each side (z_e) is calculated by Eq. (8) and is used in the subsequent analysis in exactly the same manner as (z) for the uniformly lined duct.

It is clear that if the duct is of any constant cross-sectional shape and if it is lined with several different materials, each material being in the form of a strip, having the long edge parallel to the duct axis, the equivalent impedance is:

$$\frac{1}{z_e}=\sum_{i=1}^{m}\frac{1}{\left(\frac{L}{l_i}z_i\right)}, \tag{9}$$

(where z_i = impedance of ith strip,
l_i = width of ith strip,
and m = number of strips).

SOLUTION OF DIFFERENTIAL EQUATIONS

For convenience the following four dimensionless parameters are defined:

$$\text{Dimension parameter}\quad \eta=\frac{A\omega|z|}{Lc},$$

$$\text{Phase angle parameter}\quad \mu=\frac{x}{|z|},$$

$$\text{Attenuation parameter}\quad \delta=\frac{\alpha c}{\omega}, \tag{10}$$

$$\text{Phase shift parameter}\quad \psi=\frac{\beta c}{\omega}.$$

Substituting the first two parameters of Eq. (10) in (5) the differential equation becomes:

$$\frac{\partial^2 p}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}-\frac{\omega}{c^2}\cdot\frac{1}{\eta}[(1-\mu^2)^{\frac{1}{2}}-i\mu]\frac{\partial p}{\partial t}=0. \tag{11}$$

For the purpose of discussing attenuation in a lined duct it is sufficient to consider only the case where the duct is terminated in its characteristic impedance, i.e., the case where there is no reflected wave from the terminal. Hence we assume a solution:

$$p=Be^{-\Gamma x+i\omega t}; \tag{12}$$

$\Gamma=\alpha+i\beta$ = propagation constant,
α = attenuation constant,
β = phase shift constant, and
B = constant.

Substitution of (12) in (11) leads easily to the following equations:

$$\Gamma=\frac{\omega}{c}\cdot\eta^{-\frac{1}{3}}[(\mu-\eta)+(1-\mu^2)^{\frac{1}{2}}i]^{\frac{1}{3}}, \quad (13)$$

$$\delta=[2\eta]^{-\frac{1}{3}}\cdot[(\eta^2-2\mu\eta+1)^{\frac{1}{2}}+(\mu-\eta)]^{\frac{1}{3}}, \quad (14)$$

$$\psi=[2\eta]^{-\frac{1}{3}}[(\eta^2-2\mu\eta+1)^{\frac{1}{2}}-(\mu-\eta)]^{\frac{1}{3}}, \quad (15)$$

$$z_c=\frac{\omega}{c\Gamma}i=\left[\frac{\psi}{\delta^2+\psi^2}\right]+\left[\frac{\delta}{\delta^2+\psi^2}\right]i, \quad (16)$$

$$D=264.1\alpha=0.0483\nu\delta. \quad (17)$$

The following numerical example will illustrate the uses of the formulas and curves (Figs. 2–5).
Duct lining: Johns Manville 1″ Airacoustic.
Impedance data: Beranek.[3]

$\nu=512$ c.p.s.; $r=5.3$; $x=-5.5$; $|z|=7.64$.

Duct dimensions: Square duct, 10″×10″.

$$\mu=-(5.5/7.64)=-0.72.$$

$$\eta=\frac{A\omega|z|}{Lc}=\frac{(2.54\times10)^2\times2\pi\times512\times7.64}{4\times2.54\times10\times3.44\times10^4}=4.53.$$

$\delta=0.070$ (read from Fig. 2).

$$D=(\text{db/ft.})=0.0483\nu\delta=0.0483\times512\times0.070=1.73.$$

The graphs of the "phase shift parameter" and the "characteristic impedance" will be found useful on calculations with finite lengths of duct. The necessary theory is given by Mason[4] and Sivian.[2]

SPECIAL CASES

Table I gives a list of approximation formulas for the calculation of attenuation rates in ducts.

[3] L. Beranek, J. Acous. Soc. Am. **12**, 14 (1940).

[4] W. P. Mason, Bell Sys. Tech. J. **6**, 258–294 (April, 1927).

TABLE I.

Parameter values	Physical conditions and occurrence	Approx. atten. formula (db/ft.)
$\mu=0$ $\eta<1$	Lining impedance pure resistance. Approximated in small ducts at high frequencies.	$D=1.585\left(\frac{\nu}{r}\cdot\frac{L_{in}}{A_{in^2}}\right)^{\frac{1}{2}}$
$\mu=-1$ $0<\eta<\infty$	Lining impedance pure negative reactance. Occurs in rigid ducts also approximated at very low frequencies in ducts having thin lining.	$D=0$
$\mu=+1$ $\eta>1$	Lining impedance pure positive reactance. Occurs in ducts where lining consists of rigid diaphragm backed by air space and also $\frac{A\omega x}{Lc}>1$.	$D=0$
$\mu=+1$ $\eta<1$	Same as preceding case only $\frac{A\omega x}{Lc}<1$. Note: Duct acts as high pass filter with cut-off frequency at $\nu_{\text{cut-off}}=2151\left(\frac{1}{x}\right)\left(\frac{L_{in}}{A_{in^2}}\right)$.	$D=2.240\left[\left(\frac{\nu}{x}\right)\frac{L_{in}}{A_{in^2}}\right]^{\frac{1}{2}}$
$-1<\mu<+1$ $\eta>1$	Lining impedance large, duct dimensions large, or frequency high, i.e., any combination which makes η large. Occurs at moderately low frequencies in ducts lined with commercial duct liners. In general the condition η large implies relatively poor attenuation.	$D=52.0\frac{r}{\lvert z\rvert^2}\cdot\frac{L_{in}}{A_{in^2}}$. Note: This is identical with the formula given by Beranek* derived from Morse theory.
$-1<\mu<0$ $\eta<\lvert\mu\rvert$	Any combination of system constants which makes μ positive and η small compared to μ. Corresponds to condition of relatively good attenuation in duct. Occurs in the middle frequency range in ducts of small size lined with commercial duct liners.	$D=1.118\left[\left(\frac{\nu}{r}\right)\left(\frac{L_{in}}{A_{in^2}}\right)\right]^{\frac{1}{2}}$

* See L. Beranek, J. Acous. Soc. Am. **12**, 228 (1940).

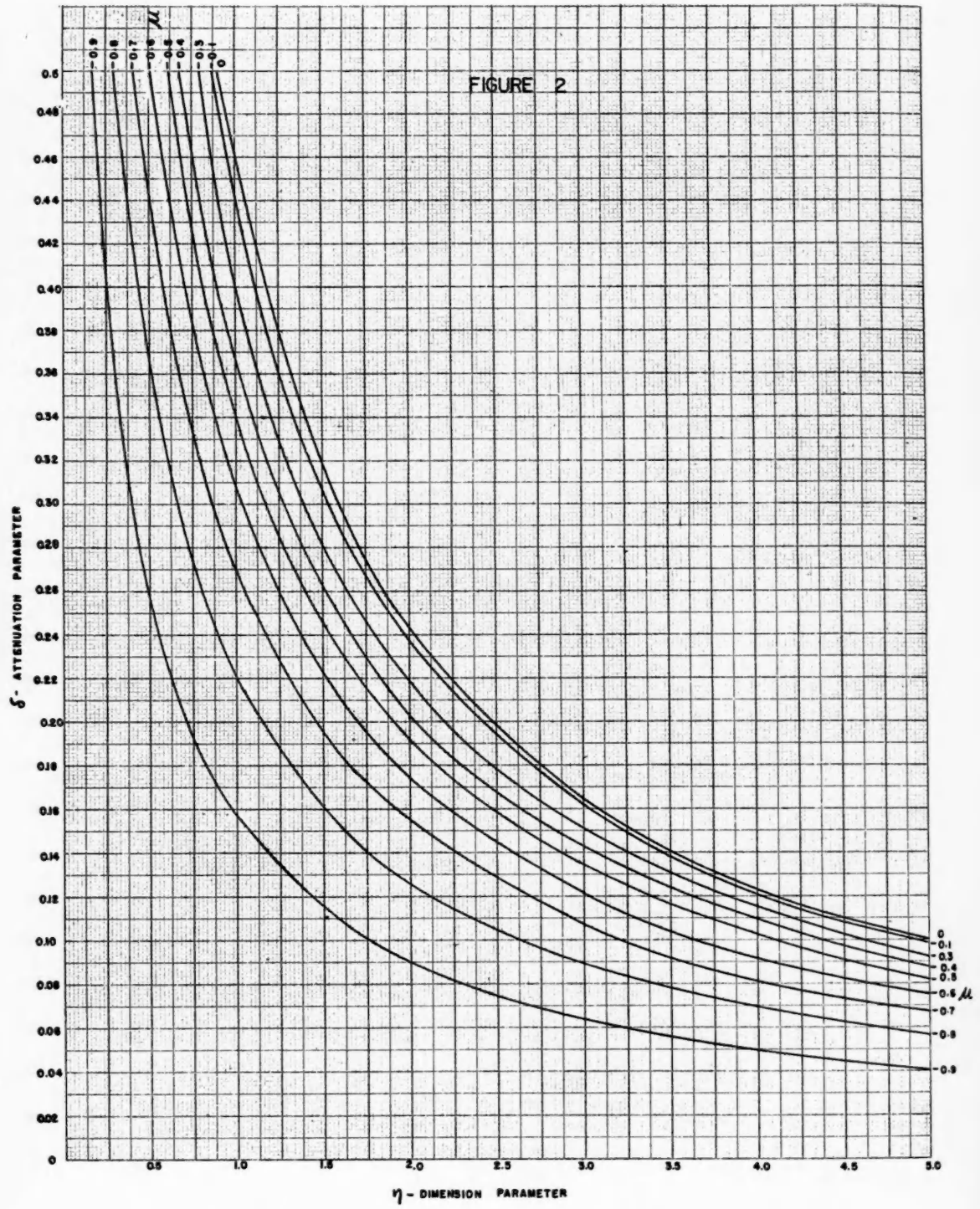

FIG. 2. Graph of δ as function of η for constant μ. $\delta=(2\eta)^{-\frac{1}{2}}[(\eta^2-2\mu\eta+1)^{\frac{1}{2}}-(\eta-\mu)]^{\frac{1}{2}}$.

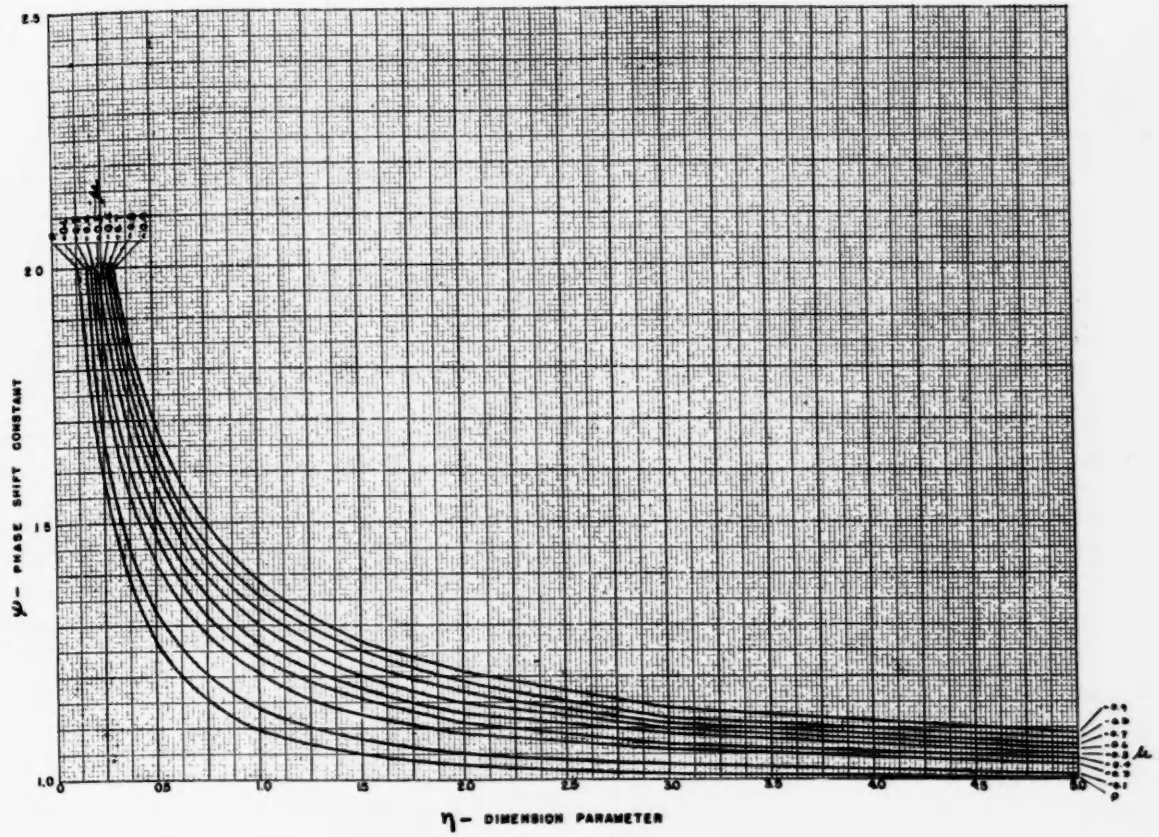

FIG. 3. Graph of ψ as function of η for constant μ. $\psi=(2\eta)^{-\frac{1}{2}}[(\eta^2-2\mu\eta+1)^{\frac{1}{2}}+(\eta-\mu)]^{\frac{1}{2}}$.

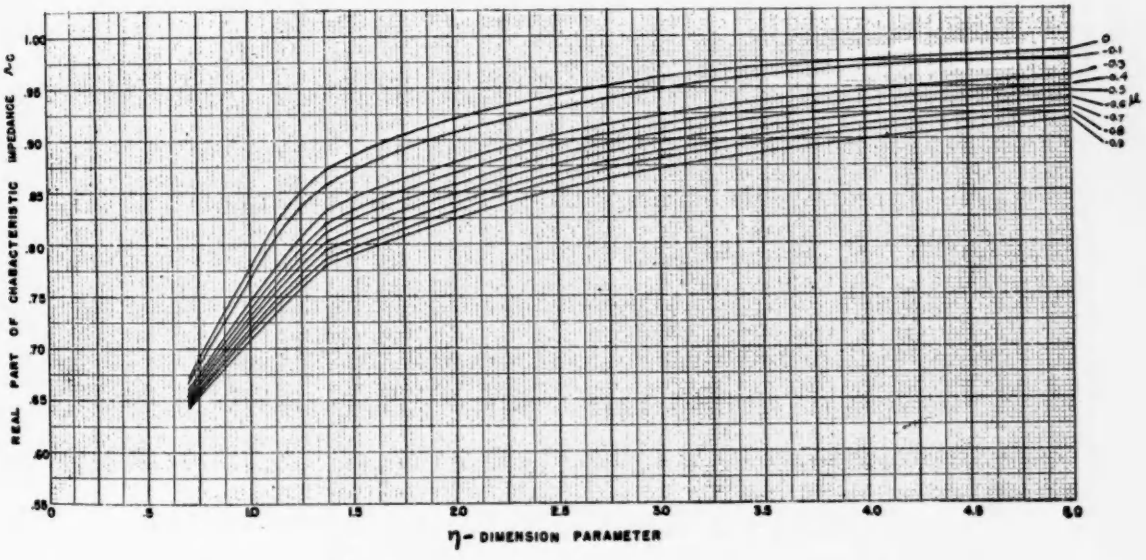

FIG. 4. Graph of real part of characteristic impedance (*ec* units) as a function of η. $r_c=\dfrac{\psi}{\delta^2+\psi^2}$.

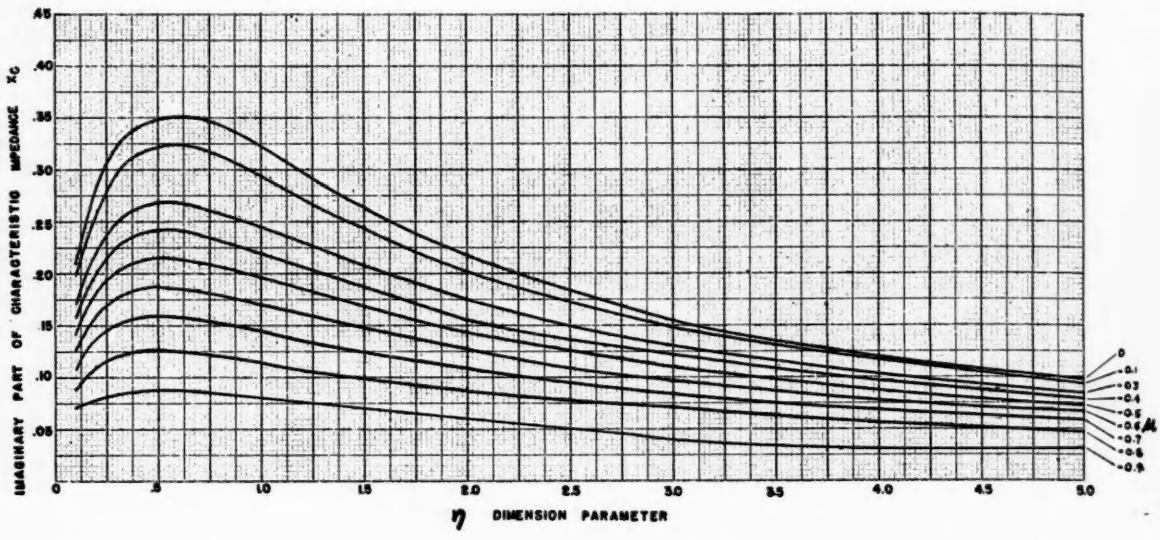

FIG. 5. Graph of imaginary part of characteristic impedance (ec units) as a function of η. $x_c=\frac{\delta}{\delta^2+\psi^2}$.

The formulas were all derived from Eqs. (14) and (17) by substituting appropriate values for the parameters and by evaluating the required limits.

ATTENTUATION AS A FUNCTION OF FREQUENCY FOR COMMERCIAL DUCT LINERS

For a given duct lined with a specific material the attenuation constant can be calculated as a function of frequency provided the lining impedance is known as a function of frequency. Such impedance measurements have been made by Beranek[3] on a number of commercial sound absorbents. A study of these data shows that a fair approximation to the impedance of rigidly backed commercial sound absorbents is given by:

$$z=r-i\cot\frac{\omega d}{c}, \tag{18}$$

where (r) is a constant independent of frequency and (d) is the lining thickness. If either the actual data or Eq. (18) is used to compute the (db/ft.) for a given duct system, as a function of frequency, a bell-shaped curve is obtained. This of course means that the attenuation in lined ducts is relatively poor at both very low and very high frequencies but becomes relatively much better in the middle frequency range. Curves corresponding to larger lining thicknesses lie above curves for smaller lining thicknesses over the whole frequency range. The frequency at which maximum attenuation occurs shifts towards the lower frequencies as the lining thickness is increased.

VARIATION OF ATTENUATION WITH DUCT SIZE

Frequently it is of interest to know how the attenuation constant varies with the cross-sectional dimensions of the duct. Examination of the table of special cases shows that for large values of η the attenuation is proportional to (L/A) and that for small values of η the attenuation is proportional to $(L/A)^{\frac{1}{2}}$. For the general case it can be shown that

$$D=\left[\frac{1+\eta_0}{\bar{\epsilon}(1+\bar{\epsilon}\eta_0)}\right]^{\frac{1}{2}}D_0, \tag{19}$$

where η_0 = dimension parameter for standard size duct,
D_0 = (db/ft.) measured in standard duct,
$\bar{\epsilon}=(L_0/A_0)/(L/A)$, and
D = (db/ft.) in arbitrary duct of dimensions (L/A).

The assumption, beyond those already made in the Sivian theory, involved in the derivation of the equation is that the lining impedance is independent of duct size.

CALCULATION OF LINING IMPEDANCE FROM ATTENUATION MEASUREMENTS

Formulas can be derived on the basis of the Sivian theory for the calculation of acoustical impedance of a duct lining from measurements of attenuation in two ducts of different dimensions each lined with the same material. The assumptions made are the same as those made for Eq. (19). The formulas are:

$$|Z| = \left(\frac{c}{\omega}\right)\left(\frac{L_1}{A_1}\right)\frac{1-\dfrac{q^2}{\bar{\epsilon}}}{\left\{\left[\delta_1^2\left(1-\dfrac{q^2}{\bar{\epsilon}}\right)+\left(1+\dfrac{q^2}{\bar{\epsilon}^2}\right)\right]^2-\dfrac{4q^2}{\bar{\epsilon}^2}\right\}^{\frac{1}{2}}}, \quad (20)$$

$$\theta = \sin^{-1}\mu, \quad (21)$$

$$\bar{\epsilon} = \frac{L_2/A_2}{L_1/A_1}, \quad (22)$$

$$q = \frac{(\text{db/ft.})_2}{(\text{db/ft.})_1}, \quad (23)$$

$$\mu = 2\eta_1\delta_1^2 - [1-(2\eta_1\delta_1)^2]^{\frac{1}{2}} = 2\eta_2\delta_2^2 - [1-(2\eta_2\delta_2)^2]^{\frac{1}{2}} \quad (24)$$

(valid only for negative μ).

Equations exhibiting lining impedance can also be derived in terms of attenuation measurements made on a single duct. Attenuation measurements are made with all four duct walls lined. The lining on two of the walls is then replaced by rigid material whose impedance is assumed to be very great compared with that of the lining. Attenuation measurements are made in this duct and from this data the acoustical impedance of the lining is calculated. The required formulas are:

$$|z| = \left(\frac{L}{A}\cdot\frac{c}{\omega}\right) \times \frac{1-2q^2}{\{[\delta^2(1-2q^2)^2+(1+4q^2)]^2-16q^2\}^{\frac{1}{2}}}, \quad (25)$$

$$\theta = \sin^{-1}\mu, \quad (26)$$

$$q = \frac{(\text{db/ft.})_{\text{two sides lined}}}{(\text{db/ft.})_{\text{four sides lined}}}. \quad (27)$$

It is to be understood that the opinions or assertions contained in this paper are the private ones of the author and are not to be construed as official or reflecting the views of the Navy Department or the Naval Service at large.

LIST OF SYMBOLS

p = Acoustic pressure (dynes/cm²).
x = Distance measured along duct axis (cm).
ρ = Air density (g/cm³).
ϵ = Acoustic displacement parallel to duct axis (cm).
t = Time (sec.).
P_0 = Atmospheric pressure (dynes/cm²).
γ = Adiabatic constant (1.41).
V = Volume (cm³).
A = Cross-sectional area of duct (cm²).
L = Duct perimeter (cm).
ϵ_n = Acoustic displacement normal to duct lining (cm).
$z = r + ix = |z|e^{i\theta}$ = acoustic impedance of lining (ρc units) $= \dfrac{\text{pressure}}{\text{linear velocity}} \times \dfrac{1}{\rho c}$.
$\epsilon_{1,2,3,4}$ = Acoustic displacement normal to duct walls 1, 2, 3, and 4 (cm).
l_1 = Width of duct walls 1 and 3 (cm).
l_2 = Width of duct walls 2 and 4 (cm).
D = Attenuation in db/ft.
z_c = Characteristic impedance of duct (ρc units).
ν = Frequency (c.p.s.).
ω = Circular frequency (radians/sec.).

Acoustical Amplification by Hearing Aids

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(Received June 1, 1944)

FROM the point of view of the wearer, the acoustical amplification afforded by a hearing aid may be defined as the difference between the sound pressure level at the ear drum when the aid is used and the sound pressure level at the same point without the aid, assuming that all other conditions are identical. Apart from the difficulty of measuring the amplification thus defined, the definition includes elements which are not inherent properties of the instrument. The fact that the acoustic output of the aid is delivered to a closed cavity, whereas, without the aid, the resonance of the open cavity of the ear affords at certain frequencies an appreciable amplification, introduces a difference which is not a property of the hearing aid.

A second definition is "the ratio expressed in decibels of the free sound field required to produce a given loudness sensation in the unaided ear to the intensity of the field at the diaphragm of the hearing aid microphone that will produce the same loudness sensation." Measurement of this ratio is the "subjective method" described by Romanow.[1] This definition is open to the practical objection that loudness matchings of two sounds presented to the ear with an appreciable time intervening is uncertain, and that the "free field" is disturbed in one case by the head of the observer and in the other by the hearing aid microphone. Further, cavity resonance operates to increase the pressure at the ear drum in the one case and does not so operate in the other.

To eliminate extraneous factors and subjective elements, the following definition is proposed: "The acoustical gain of a hearing aid is the difference expressed in decibels between the sound pressure level at the diaphragm of a standard microphone that will produce a given electrical signal in the output of a stable measuring system, and the sound pressure level, at the diaphragm of the hearing aid microphone, which will produce the same signal when the receiver of the aid is acoustically coupled to the diaphragm of the standard microphone by means of a specified closed coupler, the acoustical environment being kept constant during the course of the comparison." If it is assumed that the acoustical termination of the receiver in the coupler approximates that when the receiver is placed in the ear, then the gain thus defined will approximate the amplification of sound intensity which the hearing aid affords the wearer. In any event, by using a standard coupler, reliable and reproducible measurements can be made for the comparison of the amplification by different hearing aid units.

APPARATUS AND TEST PROCEDURE

The equipment set up for carrying out measurements on six different hearing aid units is shown in Fig. 1. A loudspeaker source of sound was set up in the angle between two sides of an absorbent lined booth 4×6×8 feet. No attempt

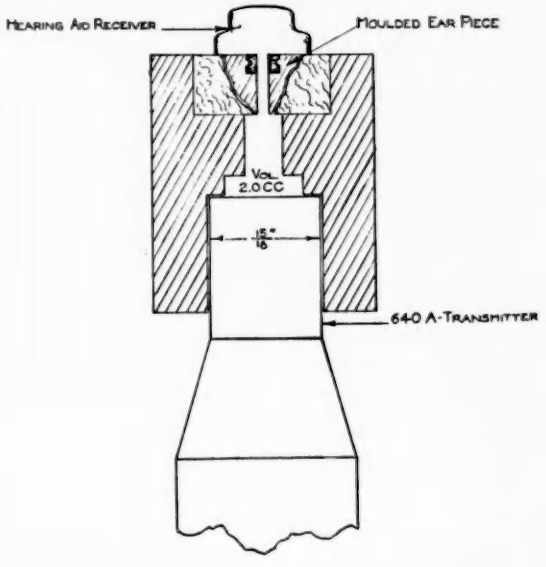

FIG. 1.

* At present at the Harvard Underwater Sound Laboratory.

[1] F. F. Romanow, "Methods for measuring the performance of hearing aids," Mono. B-1314, Bell Tel. Tech. Pub. (1940).

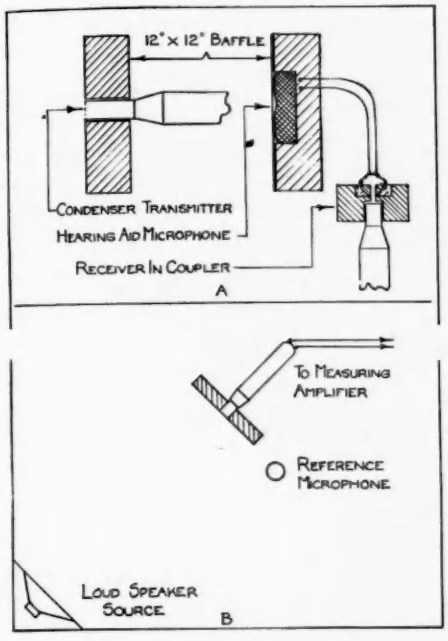

Fig. 2.

was made to secure so-called "free field" conditions since this is not a requisite under the measuring procedure used. It is required only that acoustic conditions within the booth shall not be altered between the measurements made with the standard transmitter alone and those with the hearing aid and coupler interposed. A beat frequency oscillator with warble attachment feeding a variable gain amplifier provided a sound source of controlled frequency and output level. The standard microphone was a Western Electric 640 A condenser transmitter, the voltage output of which is proportional to the acoustic pressure. The output of the transmitter was fed through an H-type variable attenuator to a stabilized amplifier, the rectified output of which was indicated on a Weston microammeter. Comparisons of acoustical pressure levels were made by comparing the attenuations needed to produce the same meter reading without the hearing aid and with it so that the amplifier operated always on the same input and linearity between the amplifier input voltage and its current output was not required.

A wooden baffle twelve inches square was mounted in a fixed position near the middle of the booth with its surface perpendicular to the axis of the loudspeaker. The cylindrical head of the condenser transmitter was closely fitted into a hole in the center of the baffle with the diaphragm of the transmitter in the plane of the baffle face. The microphone of a General Radio sound level meter was fixed in a position a few inches from one edge of the baffle, and served to determine the approximate sound intensity level at which the measurements were made. Leads from the transmitter and the monitoring microphone were carried outside the booth. With this arrangement, the output of the loudspeaker was adjusted to produce a sound level of 50 db as measured on the *B* scale of the sound level meter. The attenuator in the measuring circuit was adjusted to approximate a chosen reading on the output meter. The exact reading was obtained by a slight variation of the loudspeaker output from the original 50-db level. The new level as given by the sound level meter minus the attenuation in the measuring circuit gives what might be called the threshold level of the unaided measuring system.

A second baffle of the same size, but with the hearing aid microphone set in a recessed portion of it, and with the microphone diaphragm in the same position as was occupied by the condenser transmitter diaphragm, was substituted for the first baffle. The condenser transmitter was coupled to the receiver of the hearing aid by means of a closed coupler. Care was taken that the conditions inside the booth were otherwise identically the same as in the first measurements. The same procedure was followed to measure the threshold with the hearing aid and coupler inserted. The difference between the two thresholds thus measured gives the acoustical gain of the hearing aid as defined above for sound at approximately a 50-db level as measured by the sound level meter.

The closed coupler is shown in detail in Fig. 2. The moulded ear-piece of a hearing aid was set in a bed of soft wax filling an insert in a wooden block. The receiver of the hearing aid was snapped on to the clip of the ear piece. The sound from the hearing aid is transmitted into the cavity shown, the total volume of which was approximately 2 cc. The barrel of the condenser transmitter was fitted nicely into a hole on the

other side of the block so that its diaphragm was one wall of the cavity.

It will be noted that the method used does not measure the actual sound pressure levels, either at the face of the diaphragm of the condenser transmitter when placed in the baffle or that at the diaphragm of the hearing aid. The sound level meter served only to insure that these two pressures were the same and to give the approximate level of the field above normal ear threshold at each frequency. This was chosen at 50 db corresponding roughly to the level of quiet conversational speech, and well above the ambient noise level in the booth. The method simply balances the amplification afforded by the hearing aid by the attenuation inserted in the output of the measuring microphone.

RESULTS

Figures 3 to 10 inclusive give the results of measurements on six different instruments, all of the vacuum-tube types which have been passed as "acceptable" by the Council on Physical Therapy of the American Medical Association. The original program contemplated measurements on all instruments passed by the council, but the program was cut short by war work. Two models by each of two manufacturers are included in the six instruments investigated. Since the whole program could not be carried out, the instruments tested will be identified only as *A*, *B*, *C*, and *D*.

Most instruments are supplied with two controls, the "volume control" which regulates the amplification, and the "tone control" which is intended to vary the frequency response of the instrument. In Fig. 3 are shown curves for three settings of the tone control of instrument *A* marked *L*, *M*, and *H*, all taken at full volume setting of the instrument. There is no marked difference in the frequency response. Presumably the *L*, *M*, and *H* stand for *low*, *medium*, and *high*, but the measurements show only differences in the general level, without significant differences in the response as a function of frequency.

Figures 4 and 5 are for two different models of another manufacturer. In this instrument there is only a single control at the command of the wearer. The instrument is "fitted" to the needs of the wearer by means of permanent screw adjustments in the amplifier case. One notes a marked similarity in the frequency response of the two models. Model 2 shows, in general, about 3 db more gain than model 1. In both models, the difference in the screw adjustments makes about 10 db more difference in the low than in the high frequency amplification.

Figure 6 shows the effect of varying the B voltage in an instrument having only a single control. In these measurements the tone control was set 10 db below the maximum. It is to be noted that in the most important range for speech, 512 to 4096 cycles, the additional gain from the 45-volt battery is only 10 or 12 db over that with the 18-volt battery. In Fig. 7, the amplifications given by the same instrument with the volume control set for full volume and for half-volume using the 30-volt battery are shown. It appears that, with the 30-volt B battery alone, as great a range of amplification is at the wearer's disposal as is afforded by the B-voltage variation from 18 to 45 volts.

The graphs in Fig. 8 are for another model by the same manufacturer. This model has two controls. The graphs show that the maximum variation in frequency response is an average reduction of about 8 db below 2048 cycles with an increase of 7.5 db in the octave above this frequency. It is possible that this differential in gain between high and low tones would be an advantage in certain types of deafness, and in adjusting the instrument for use in a noisy environment. The testimony of the wearer on this point would be of more value than the opinion of a physicist.

Figure 9 shows the range of variation in response and in volume possible with hearing aid *D*. Here we note a fairly uniform response over the entire speech range, with the ability to produce marked suppression of low pitched sounds with relative small reduction in the gain for high pitched sounds. It is interesting to note that this degree of flexibility is not possible with the low gain setting of the volume control. The shape of the curve for minimum volume and minimum tone is the same as that for full volume and full tone rather than for full volume and minimum tone.

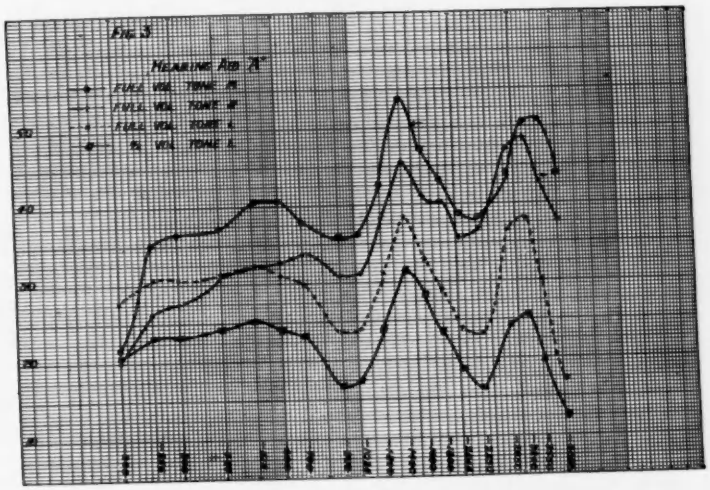

FIG. 3.

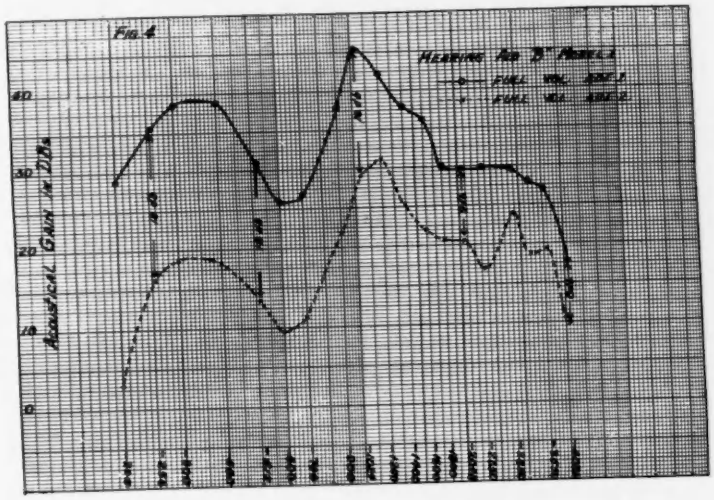

FIG. 4.

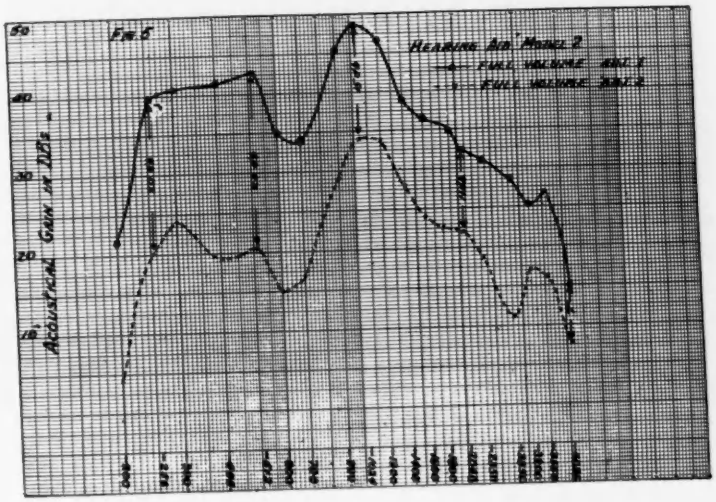

FIG. 5.

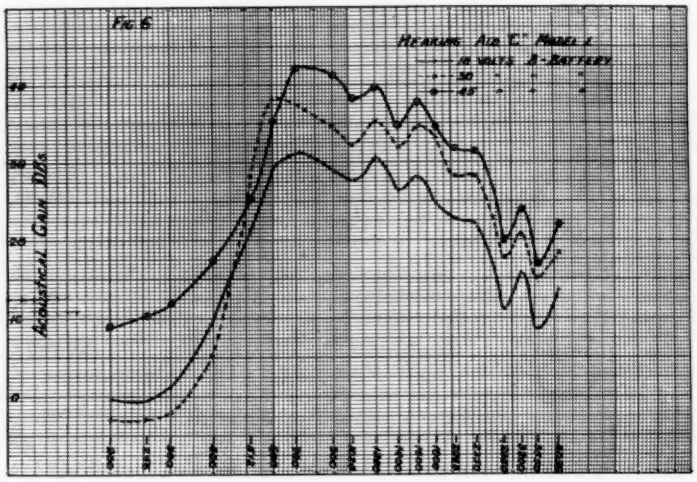

Fig. 6.

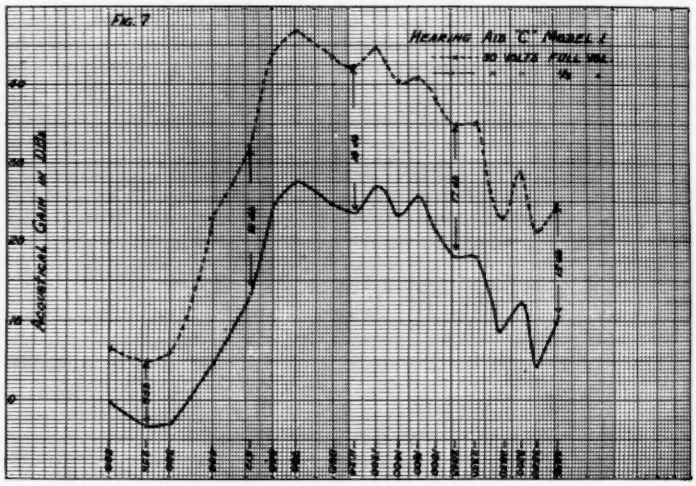

Fig. 7.

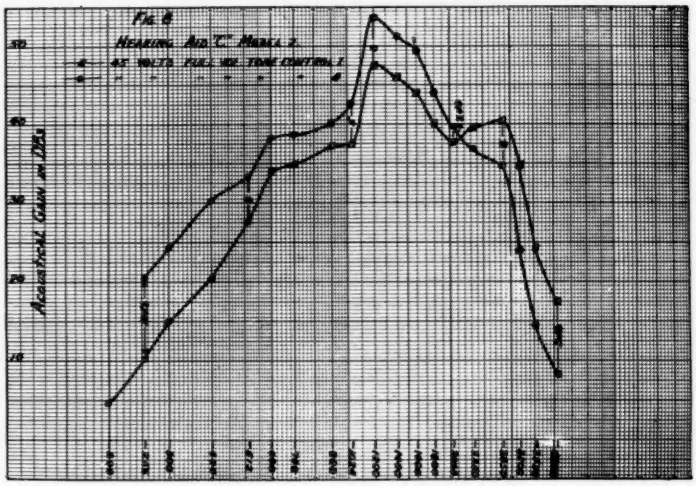

Fig. 8.

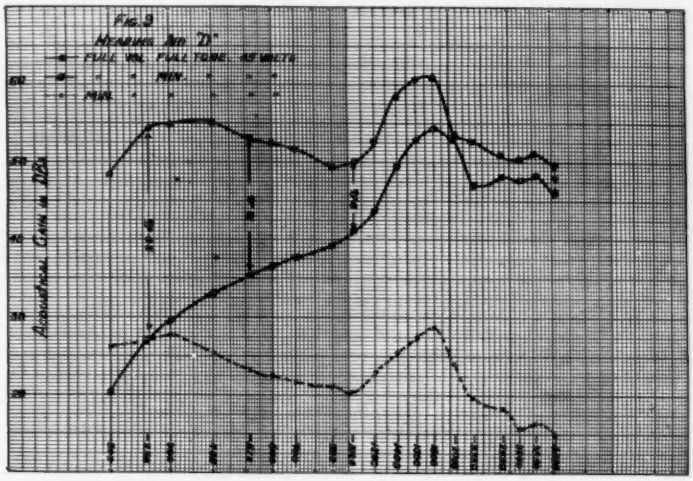

FIG. 9.

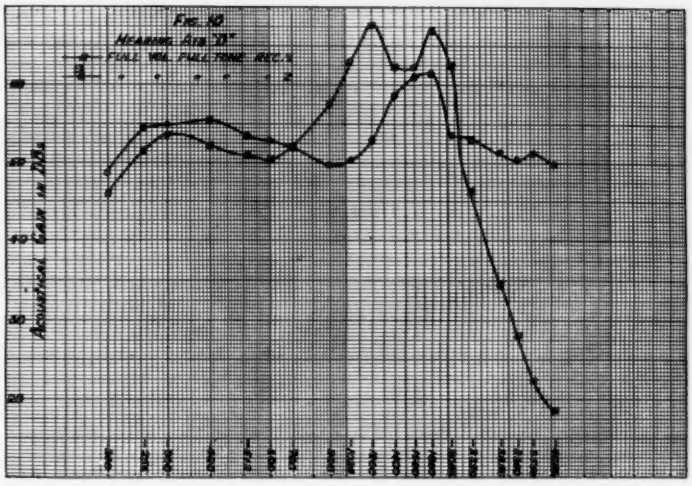

FIG. 10.

Figure 10 shows the effect of the frequency characteristics of the receiver on the over-all response. The two curves are for the same instrument as shown in Fig. 9, but with two different receivers. These curves bring out a fact which I think is generally true, that in the present state of the art, the electroacoustic transducers which terminate the amplifying system are the least controllable factors in hearing aid performance.

OUTPUT *VS.* INPUT AS A FUNCTION OF INPUT LEVEL

All of the foregoing data were obtained at an input level of approximately 50 db as measured on the *B* scale of the General Radio sound level meter. In Fig. 11 are shown the relations between the acoustical pressure levels at the microphone of hearing aid *C*, model 2 and the pressure levels set up in the coupler for increasing values of the former. These measurements were made using pure tones at the fixed frequencies shown. The sound pressure level as given by the sound level meter is plotted horizontally and the levels set up in the coupler are shown on the vertical scale. For the straight portions of the graphs the amplification is constant, but above a certain input level the relation between input and output ceases to be linear, indicating the limit of the power capacity of the hearing aid. Thus at 1024 cycles this particular instrument begins to overload at an input level of 55 db and shows a maximum level which it can produce of about 102 db above normal ear threshold. The wave form at 512 cycles as shown by a cathode-ray

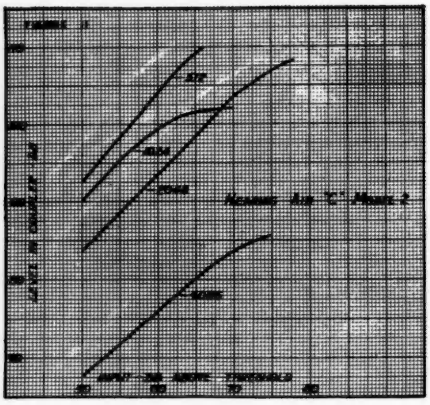

FIG. 11.

oscilloscope was distorted at input levels above 50 db. At the higher frequencies, no marked distortion was apparent, even at input levels which overloaded the hearing aid amplifier.

Time did not allow complete measurements of this sort on all the instruments, but enough measurements were made to indicate that the results shown in Fig. 11 are fairly typical of all the instruments tested. The results suggest that in a standardized procedure for measuring the gain of hearing aids, the input levels should be specified and should be below the overload level of the instrument.

The data presented do not by any means cover all the information that is desirable in evaluating the merits of a particular hearing aid. They do, however, present a fairly simple method of measuring the acoustical gain. As will have been noted, the method is essentially that which Romanow[1] has called a "coupler calibration," without imposing the somewhat difficult condition of setting up a "free field." It is hoped that it may serve a useful purpose in helping to standardize the test procedure for determining the acoustical gain of hearing aids.

It is a pleasure to acknowledge the cooperation of the Council on Physical Therapy of the American Medical Association in supplying the instruments and a part of the equipment for the foregoing series of measurements.

Practical Hearing Aid Measurements

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Sonotone Corporation, Elmsford, New York
(Received June 15, 1944)

INTRODUCTION

IT is the purpose of the authors to indicate in this paper the measurements made on hearing aids in production and their basis in standard laboratory tests. The frequency response curves desired in an instrument should take into account the method in which an instrument is worn. The effects of the body are shown and data are given on a simple artificial body which has shown characteristics similar to an average group of users on several instruments.

The purpose of a hearing aid is to increase the apparent sound intensity heard by a hard-of-hearing individual, so as to provide increased intelligibility and enjoyment. To translate this conception into laboratory measurements is difficult, but effective methods have been devised to measure the increase of sound pressure caused by an aid. Some of these methods have been well described by Romanow.[1]

The measurements of hearing aids require:

(a) A sound booth.
(b) A sound source.
(c) An artificial body.
(d) An artificial ear (or mastoid for bone conduction).

These will be described as they apply to both production and research.

GENERAL LABORATORY FACILITIES

Sound Booth

A soundproof booth which is sufficiently large and non-reflecting to permit free-space measurements at distances of at least two or three feet is an obvious necessity. The constructional details of one of our sound booths are illustrated in Fig. 1.

The sound source or artificial voice is located near one corner. Ozite baffles around the walls minimize direct reflections of sound. Because of utilizing only the speech range, fewer baffles are required than in the booths described by Olson.[2] The use of Ozite baffles in the booth reduces reflections sufficiently to allow working at approximately twice the distance from the source that was possible without them.

Velocity microphone measurements, with the ribbon directed first towards the source and then perpendicular to it, show that reflections are approximately 20 db weaker than the direct wave at 1 foot from the source and 15 db weaker at 2 feet. A distance of 2 feet is generally used.

Frequency Generating and Measuring Equipment

In taking frequency characteristics of hearing aids, care must be exercised to avoid acoustic overload. This may be ensured by making a continuous inspection of the output wave form. This is done by connecting the horizontal axis controls of an oscilloscope to a signal generator, and the vertical axis controls to the output of the hearing aid through an amplifier and potentiometer network.

A photograph of such a set-up is shown in Fig. 2. The schematic diagram is shown in Fig. 3.

In this device, a drum is turned simultaneously with the signal-generator frequency-control dial.

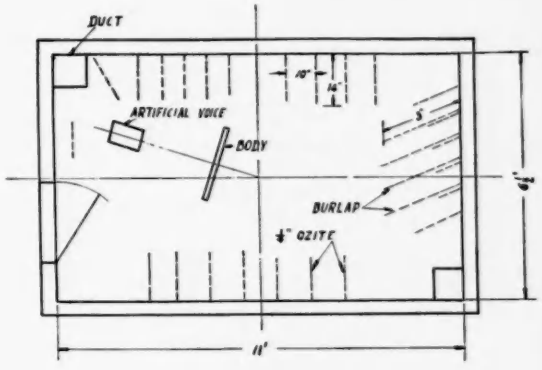

FIG. 1. Soundproof booth construction.

[1] F. F. Romanow, J. Acous. Soc. Am. **13**, 294 (1942).

[2] H. F. Olson, J. Acous. Soc. Am. **15**, 96 (1943).

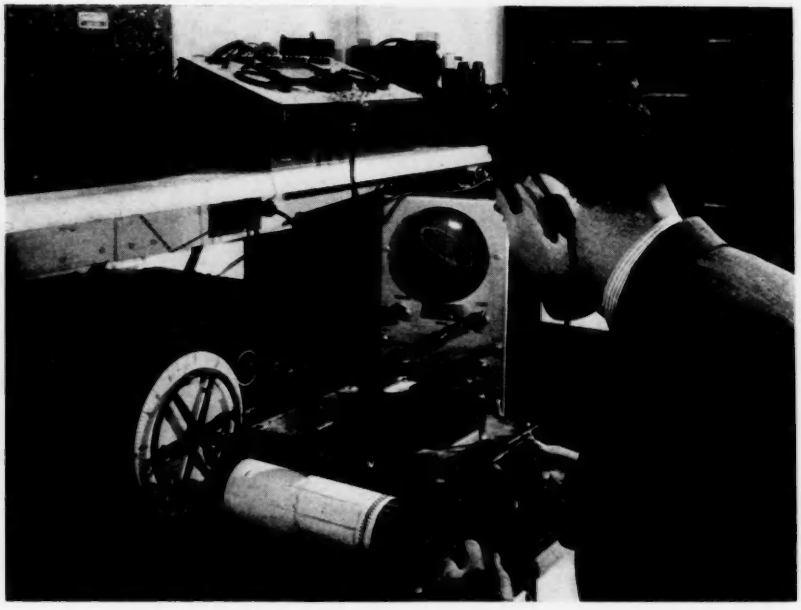

FIG. 2. Photograph of manual curve drawing equipment.

The other operating handle is manipulated to keep the oscilloscope figure at a constant height. The cable controlling the logarithmic attenuator and pen causes a curve to be automatically traced. A meter may be used, as shown in the photograph, when linear recording is desired.

If distortion occurs, the ellipse becomes distorted by an amount which can be visually estimated.

Automatic motor-driven equipment is now generally used for this curve tracing. The visual inspection feature of the oscilloscope is also used when this equipment is employed.

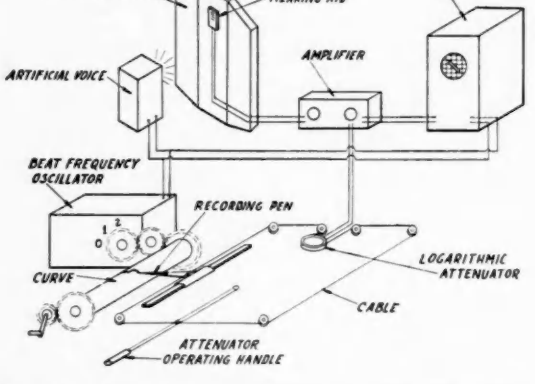

FIG. 3. Schematic layout of curve drawing equipment.

If a phase shifting network is introduced into the amplifier network, the ellipse may be collapsed into a straight line, provided no distortion exists. The amount of distortion may then be estimated by noting the relative amplitude perpendicular to this base line, and the order of the harmonic is evaluated by counting the number of loops along the line.

Artificial Voice

The artificial voice used in the booth consists of a dynamic speaker modified so as to be extremely light and flexible, but inertia controlled, as suggested by Rice and Kellogg[3] and by Olson.[4]

The suspension system consists of a chamois leather rim and a string suspension through the voice coil cylinder, which is ventilated to avoid acoustic stiffness. The diaphragm is constructed of 0.0027-inch paper which has been Bakelite impregnated. The voice has a stiff 110° closed-center (one piece) folded cone 2.55 inches in diameter. A doubly conic reinforcing ring is cemented over the fold to stiffen that region and

[3] E. W. Kellogg, Dynamic Loudspeaker Patent No. 1,795,214; C. W. Rice and E. W. Kellogg, Dynamic Loudspeaker Patent No. 1,894,197.

[4] H. F. Olson and F. Massa, *Applied Acoustics* (The Blakiston Company, Philadelphia), p. 203.

thus extend the high frequency range. See Fig. 4.

The mass of the cone and voice coil assembly is 1.43 g. The calculated air mass on a piston of this diameter radiating into air is approximately one-seventh of this value.

The voice is mounted in a box 7×12×7.5 inches, which was made large in order to keep the acoustic stiffness low. The enclosure is internally damped with soft felt baffles.

The front grille is lined with several layers of soft cloth. This introduces some damping in the working range and improves the wave form.

The response of this voice is shown in Fig. 5. These curves were taken with a 1-inch condenser microphone, the free-field calibration for which is shown at the bottom. Inspection of these curves, one of which is taken at 1 foot and

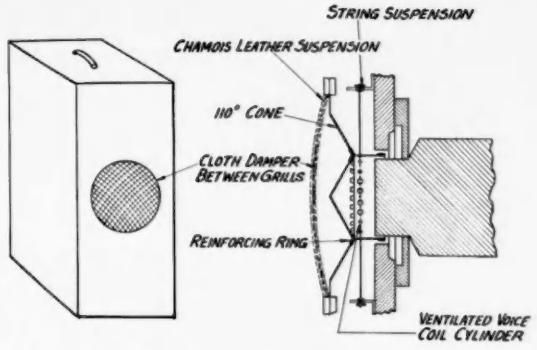

FIG. 4. Construction of free-space artificial voice.

the other at 2 feet separation from the artificial voice, shows that, since the pressure is substantially 6 db less at 2 feet than it is at 1 foot, the intensity is reduced in proportion to the distance from the source. This is the characteristic of a point source.

Artificial Body

In general, the response of a hearing aid suspended in free space is not the same as that of an instrument worn on the body. The effect of the body will differ, depending on the hearing aid being measured, the individual, the clothes worn, and the position in which the hearing aid is worn.

The difference between the observed output when the instrument is worn, and the output of the same instrument suspended in space at the

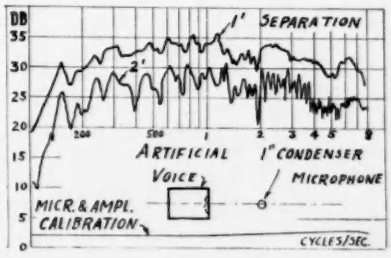

FIG. 5. Curves of artificial voice using 1-inch condenser microphone.

same position, is shown in the upper group of curves of Fig. 6. This curve is characteristic of the instrument used.

There is a 5-db increase up to 1000 cycles when a sample hearing aid is worn outside the clothing on an average person, but the high frequency response is substantially unchanged. A considerable dip occurs around 1500 cycles. The dip for each individual is quite narrow, and is increased when the microphone is lifted slightly from the person. The extremes of variation of various individuals from the average of the full line is indicated by dots.

The losses due to clothing are further illustrated by the curves in Fig. 7, which show two illustrative examples of the effect of wearing a hearing aid under a coat, as in a shirt or vest pocket. The output of the hearing aid when suspended in free space is taken as a reference. The loss at 1000 cycles averages 7 db and losses at higher frequencies progressively approach 20 db.

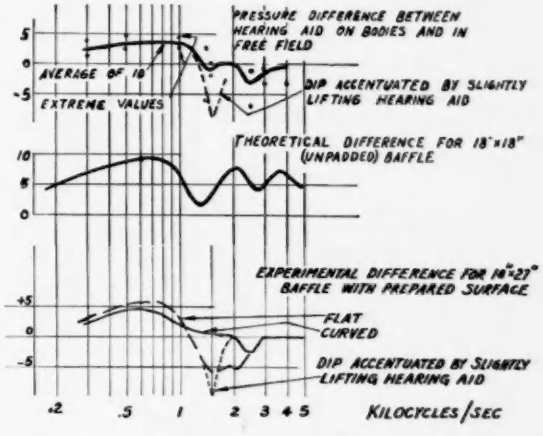

FIG. 6. Curves showing differences between pressures on bodies and free field when using hearing aid.

It is evident from the curves in Figs. 6 and 7 that all the conditions under which a hearing aid is expected to operate cannot be simultaneously simulated, for in the laboratory the best that can be done is to approximate the optimum conditions of operation and construct an artificial body that will simulate these average conditions.

The alternate increment and diminution in increased output suggests that the diffraction pattern of the body and the pressure increase on the microphone due to the body are not simple pressure doubling effects, but resemble the behavior of baffles as described by Sivian and O'Neil[5] and Muller, Black, and Davis.[6]

These investigators have shown that, for normal incidence, the thickness of the baffle has no effect upon the pressure increase on the front surface. It is, therefore, plausible to assume a flat baffle for an artificial body. The periodicity of the human body indicates that an 18-inch square baffle might be a fair approximation, although this periodicity is not as marked as in the baffles of Muller, Black, and Davis. The departure is due partly to the curvature of the front of the body and its greater length, and partly to the absorption of the clothing.

The practical construction of an artificial body is shown in Fig. 8.

To make the artificial body simulate the human body, the front is curved slightly, the vertical dimension is increased to 27 inches (while keeping the hearing aid near the top), the flesh is simulated by a ½-inch layer of sponge rubber, and common clothing is added (an undershirt, shirt, and jacket). Dissipation equivalent to the average chest and clothing is secured by the use of a ½-inch thick soft sponge rubber pad under common clothing. (It is not necessary to use sponge rubber over the entire surface.) This smooths out the pressure response differences due to the baffle above 800 cycles and increases the correspondence to the average human body.

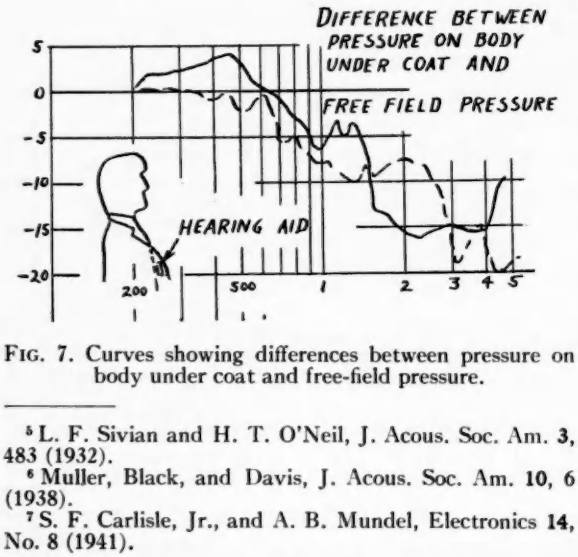

FIG. 7. Curves showing differences between pressure on body under coat and free-field pressure.

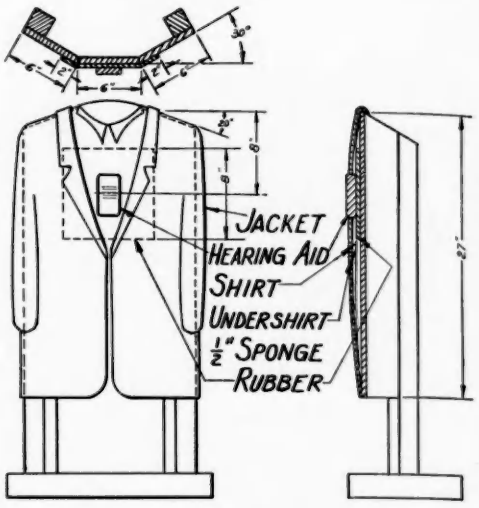

FIG. 8. Artificial body construction.

The experimental results on the complete artificial body are shown in the solid lower curve of Fig. 6. This has the same response increase up to 1000 cycles, the same dip at 1500 (as a function of the closeness of the hearing aid to the artificial body, which is characteristic of human bodies), and the same dip at 2500 cycles. Analogous curves on human bodies and on this artificial body using other types of microphones show equally good correspondence. However, the curves of output increment have a different shape, depending on the hearing aid used, and it has been found necessary to take hearing aid curves on such a representative body to ascertain the true frequency characteristics of an instrument under service conditions.

The need for sloping the sides of the baffle backwards is demonstrated by the lower dashed curve of Fig. 6. The pressure increase in the low frequency region is too great, and too wide a dip occurs around 1500 cycles. By sloping the

[5] L. F. Sivian and H. T. O'Neil, J. Acous. Soc. Am. **3**, 483 (1932).
[6] Muller, Black, and Davis, J. Acous. Soc. Am. **10**, 6 (1938).
[7] S. F. Carlisle, Jr., and A. B. Mundel, Electronics **14**, No. 8 (1941).

sides back at an angle of 30°, satisfactory correspondence to the average human body is secured. The upper corners of the baffle have been sloped 20° so as to allow the clothing to hang smoothly.

Artificial Ear

For "insert" receivers, standard artificial ears of 2-cc volume are used. These are closed couplers which closely approximate the ear and ear insert into which the receivers work. These have become standardized and are referred to by Romanow.[1]

Artificial Mastoid

An "artificial mastoid" is used for testing bone conduction receivers, and is so called because bone "units" are generally worn over the mastoid bone. A device of this type has been described by Hawley.[8] Other devices, suitable for production comparison of bone units, will be described later in this paper.

Artificial mastoids may be calibrated by subjective methods. To do this:

1. Comparative measurements are made adjusting bone conduction receivers to have the same subjective loudness as calibrated air conduction receivers.
2. A correlation is made between this subjective loudness obtained when a bone unit is held on the head, and the output from the artificial mastoid when the same bone unit is placed thereon and the same voltage applied.
3. A velocity calibration of an artificial mastoid may then be made by taking velocity measurements on such calibrated bone units.

So far as the authors know, no exhaustive studies on artificial mastoids have been published which show a simple correspondence between subjective loudness and the artificial mastoid output for more than one type of bone conduction receiver. The work becomes protracted because no one type of bone unit may be used to calibrate an artificial mastoid for use with any except that type of bone unit.

[8] M. S. Hawley, Bell Lab. Rec. **18** (Nov. 1939).

PRODUCTION TEST EQUIPMENT

Since hearing aid components are small and intricate, it has been found advisable to check every production component as well as each completed instrument.

Frequency Generating and Detecting Equipment

The source of voltage used for this could be a set of fixed oscillators, or a sweep-frequency oscillator with manual control. Testing has been greatly expedited, however, by the system schematically illustrated in Fig. 9. The frequency of a beat oscillator is varied from low to high frequency by an automatic electronic circuit, which simultaneously controls the horizontal position of a cathode-ray oscilloscope beam. The vertical deflecting plates are connected to the output of the particular hearing aid or component being measured. A continuous frequency pattern is then displayed to the test operator, who checks the over-all frequency response of the instrument, varies the tone and volume controls, and notes the overload characteristics.

The principle of the control tube and its affiliated circuit is the same as that used in automatic radio tuners, in which the effective inductance of a circuit is made a function of the potential applied to a grid. This is in turn controlled by a saw-tooth oscillator.

This system has been described by S. F. Carlisle, Jr., and A. B. Mundel in *Electronics*,[7] in which this illustration appeared.

The appearance of a typical projected pattern,

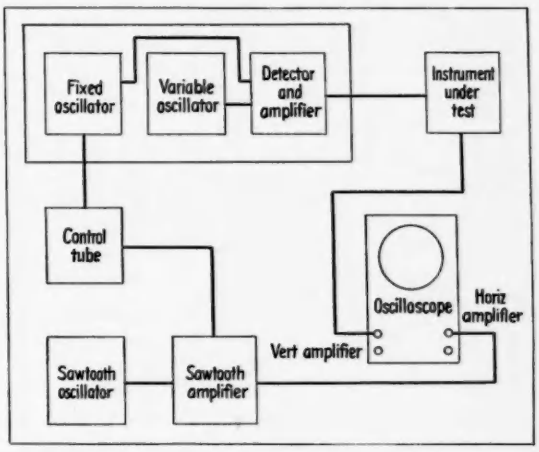

FIG. 9. Schematic diagram of electronic curve projector.

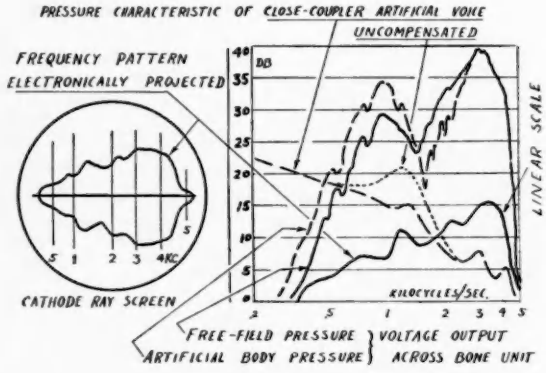

FIG. 10. Comparison of electronically projected frequency pattern with free-field pressure, and calibration of close-coupler artificial voice.

taken on production test equipment, and traced from a cathode-ray screen, is shown at the left of Fig. 10. The ordinate is the linear voltage output of the hearing aid, measured across a bone unit used as a load.

Sound Source—Close Coupler with Body Baffle Calibration

The artificial voice to which this instrument was coupled in order to secure this pattern is shown diagrammatically in Fig. 11.

Placement of each instrument in the sound field as done in the laboratory gives characteristic true-to-service conditions, but requires too much space to be applicable to each production instrument and is accordingly utilized only for standardization.

Placement of instruments in a limited field, i.e., in a small, acoustically deadened compartment, with a loudspeaker unit at one side, was used for a while but was found to be subject to the effects of noise. The close coupler illustrated in Fig. 11 has, therefore, been designed.

The mechanism is similar to that of a simple bending-reed type soundpower telephone, with a corrugated beryllium copper diaphragm. It is damped by thin cloth pads cemented over holes in the cavity behind the diaphragm, and the front chamber is further damped by cloth cemented over apertures in it.

A dynamic receiver could be designed or a standard flat receiver could be compensated to give an equally efficacious result. However, a fairly large diaphragm should be used, in order that the low frequency response of the system should not be too critical to the seal between the voice and the face of the hearing aid.

The dotted curve of Fig. 10 shows the uncompensated frequency characteristic of this close coupler. The 1300-cycle peak is largely eliminated by a simple electrical network, the final characteristic being indicated by the dashed line.

The exaggeration of low frequencies is accomplished by matching impedances in that region.

In order to show the comparison of this pattern with characteristics taken on the previously described laboratory equipment, this pattern has been transposed to the same frequency axis as other curves on the right of the figure, retaining the linear vertical axis.

At the top of this group of curves, the logarithmic frequency characteristic of the same hearing aid taken on the previously described laboratory equipment is shown with a solid line in a free field and with a dashed line on the artificial body. This is the voltage across a bone unit load, as used before.

It may be seen that the rapid fall of the curves below 1000 cycles (which is due partly to the impedance characteristic of the bone unit) would make observation of low frequency response impossible on a linear oscilloscope unless lows were exaggerated in the close-coupler voice.

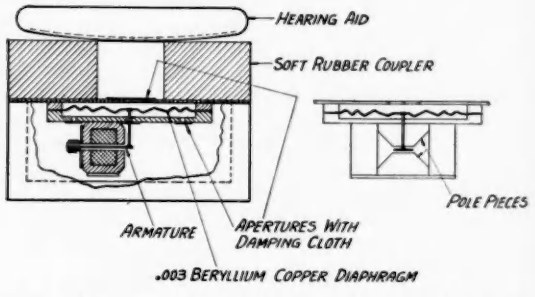

FIG. 11. Construction of close-coupler artificial voice.

Air and Bone Conduction Receiver Measuring Devices

All production earphones are tested on a standard artificial ear, and all production bone conduction receivers, or bone units, are tested on artificial mastoids.

The device shown diagrammatically in Fig. 12

is analogous to that described by M. S. Hawley.[8] The bone unit is pressed against a stiff and vibration-absorbent rubber pad to simulate the head impedance, and the velocity of the probe contacting the bone unit actuates the armature in the magnetic system.

The work of G. V. Békésy[9] and that of Ernst Bárány[10] indicates that the mechanical impedance of the head is largely stiffness and resistance up to 1000 cycles. The magnitude determined by them in the 500- to 1000-cycle range is 1 to 2×10^8 dyne/cm stiffness and 1 to 2×10^4 dyne sec./cm viscous resistance. In the electrical sense, these should be regarded as a capacitance and resistance connected in series.

We have found the head to be fairly well simulated by a $\frac{1}{8}$-inch thick pad of Hewlett oil-resisting rubber, which is rather stiff and vibration-absorbent. It should be shaped to fit the contacting surface of the bone unit.

The magnetic system, which is similar to that used in the close coupler, is flexibly suspended in order that pressure on the drive rod will not cause closure of the lower air gap. The drive rod is mechanically supported along its length by sponge rubber washers.

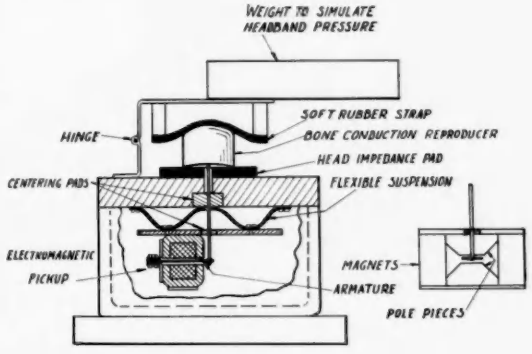

FIG. 12. Construction of probe-type artificial mastoid.

[9] G. V. Békésy, Ann. d. Physik **13** (1932).

[10] Ernst Bárány, "A contribution to the physiology of bone conduction," Acta-Oto-laryngologica Supplement XXVI (Appelbergs Boktryckeriaktiebolag, Uppsala, Sweden, 1938), pp. 56–64.

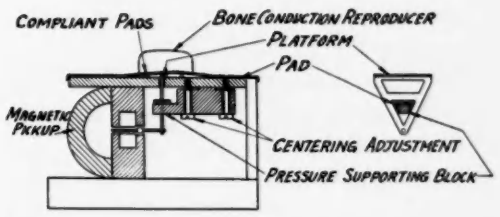

FIG. 13. Platform type artificial mastoid construction.

A pressure equivalent to a head band is applied by a soft rubber strip and weight.

An alternative type of artificial mastoid is shown in Fig. 13.

This one is similar to the other except that, instead of a probe, a platform is used on which to rest the bone unit. This platform is part of a small aluminum member which transmits an average velocity, taken across the base of the bone unit, to the magnetic pick-up armature. Rubber pads are used to simulate the flesh and to provide stiffness.

A possible advantage of this type over the probe type is that it can be used to measure the average velocity of the face of a bone unit, which is not necessarily uniform.

For production use, less damping is used in these devices than is representative of an average head. This facilitates the recognition and adjustment of the normal mechanical resonances in the bone units.

CONCLUSIONS

The need for using an artificial body for hearing aid measurements has been demonstrated, together with the application of an electronic method of continuously projecting frequency characteristics in hearing aid production.

ACKNOWLEDGMENT

We are indebted to Dr. Fred Kranz, Dr. Emil Greibach, Mr. Harry Pearson, Mr. P. R. Werner, Mr. S. F. Carlisle, Jr., Mr. H. A. Anderman, and other members of the Sonotone Corporation engineering staff for collaboration in the general development.

Acoustic Sound Filtration and Hearing Aids*

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(Received June 13, 1944)

THE receiver of a hearing aid is usually connected with the ear by means of an ear mold made of Lucite to fit the external ear canal. Littler[1] observed that the mold may be responsible partly for the weak high frequency output of hearing aids. Later it had been observed that a change of the dimensions of the sound conveying canal as well as a change of the length of the tip may bring about a change of quality of sound transmission. A small tube with a side branch, an acoustic high frequency pass filter, has been introduced also in the clinical art of fitting hearing aids.[2,3]

It is the purpose of this paper to study experimentally variations of the acoustic line between the hearing aid receiver and the drum and to supply a mathematical analysis for the phenomena involved.

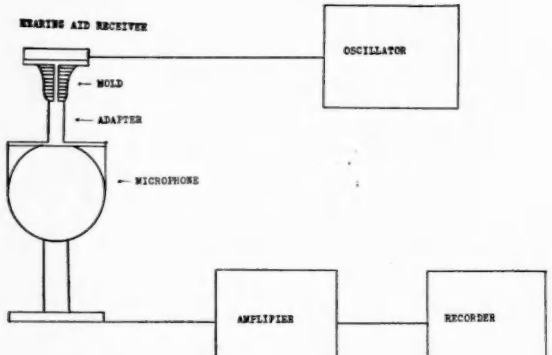


FIG. 1. Diagram of experimental set-up.

EXPERIMENTS

Four molds were used for the study, two being conventional molds with sound conveying canals having a diameter of approximately 2 to 3 mm, a third mold had a canal diameter of 1½ mm, and the fourth mold had a diameter of 5 mm. The lengths of the canals were 20 mm, except for the wide canal which was 18½ mm long. The high pass filter used between mold and receiver was a brass tube 5 mm long with a diameter of 4 mm. The side branch was a hole 1 mm long, having a diameter of 1.2 mm. Two magnetic receivers were used for the experiments, receiver I had a high peak near 1500 c.p.s. with rapidly declining output above the peak and receiver II, a Western Electric Hearing aid receiver No. 714, a so-called *C* receiver, had a fairly flat response curve up to 6000 c.p.s.

The receiver was connected to an oscillator (Fig. 1). The frequency output of the oscillator was varied continuously by use of a motor drive. The output voltage was constant. The receiver was connected to the ear mold under test which in turn was coupled to a dynamic microphone by an adapter consisting of a tube 10 mm long and 6.34 mm in diameter. The output of the receiver was picked up by the microphone and fed to an amplifier and thence to a high speed level recorder.

The curves in Fig. 2 show the response of the receiver I connected to the molds only, on the left side of Fig. 2 and connected to the molds with insertion of the high pass filter on the right side. The narrow mold shows a broader peak than the other two obliterating the sharp peak at 1500 c.p.s. This is what might be expected since the resistive component of impedance in the canal of 1½ mm diameter is larger than in the other two. The conventional and wide mold response curves do not exhibit significant differences because these occur in the higher frequencies

* It is to be understood that the opinions or assertions stated in this paper are the authors' own and are not to be construed as official or as reflecting the views of the Navy Department or the Naval Service at large. This paper was read at the twenty-ninth meeting of the Acoustical Society of America, May 12–13, 1944, New York, New York.

† Physicist of the Material Laboratory of the Navy Yard, New York, New York.

[1] T. S. Littler, J. Sci. Inst. **13**, 144 (1936).

[2] T. H. Halsted and F. M. Grossman, N. Y. State J. Med. **42**, 1944 (1942).

[3] F. M. Grossman, Arch. Otolaryngol. **38**, 101 (1943).

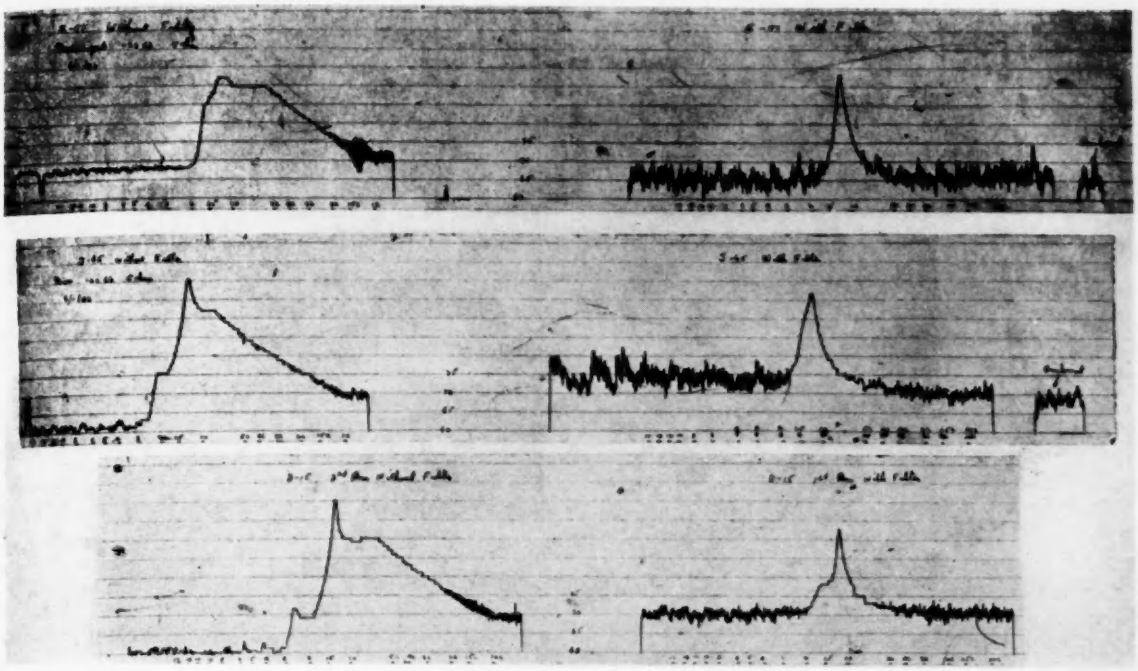


FIG. 2. Frequency response curves of hearing aid receiver I, connected to different molds with and without filter. Top curves: narrow mold. Middle curves: conventional mold. Bottom curves: wide mold. The horizontal lines on the graphs are five decibels apart.

which are not produced by the receiver under test.

The curves taken with the filter in place show for all molds that the low frequencies are reduced. The jagged lines on either side of the peaks are probably ambient since the hole in the filter will permit sound from the outside to pass into the system.

In order to investigate the acoustic line under discussion at higher frequencies receiver II was chosen. Figure 3 shows difference curves obtained from response curves of receiver II which was connected to the molds with different diameter and the conventional mold with the filter inserted. The zero line is response curve of receiver II with the conventional mold. The other curves are response curves given in plus and minus differences, respectively, from the response with the conventional mold. The narrow mold shows a considerable difference in the range between 1300 and 5000 c.p.s. The wide mold differs in the range of 2500 to 5000 c.p.s. from the conventional mold. Most remarkable is the comparison between conventional mold without filter and conventional mold with filter inserted. The response with filter shows uniformly weaker output of 15 to 18 db up to 1000 c.p.s. From there the curve approaches the x axis. Between 3000 and 4000 c.p.s. the output of the receiver with filter inserted is slightly higher than that without the filter. In terms of insertion loss this part represents negative insertion loss. The response curves obtained from receiver II connected to two conventional molds in succession are in close agreement with each other.

ANALYSIS

I. Insertion Loss Due to Filter

δ = insertion loss due to filter (db),
v_3 = particle velocity at entrance to ear mold, filter inserted in cm/sec.,
$\bar{v}_3$ = particle velocity at entrance to ear mold, filter absent in cm/sec.,
v_b = particle velocity at entrance to filter in cm/sec.,
ρ = density of air in g/cm,
c = velocity of sound cm/sec.,
ω = circular angular frequency in radian/sec.,
ν = frequency in c.p.s.,
p_0 = pressure developed in neck of receiver if terminated by infinite impedance (dynes/cm^2),

p = pressure actually existing inside filter in dynes/cm,
z_h = impedance of filter hole in ρc units
$$= \frac{\text{pressure}}{\text{linear velocity} \times \rho c},$$
z_s = impedance of source looking into the neck of receiver in ρc units,
z_5 = impedance in filter at position (5) in ρc units,
z_4 = impedance in filter at position (4) in ρc units,
$z_3 = r_3 + ix_3$ = impedance at entrance to ear mold in ρc units,
z_2 = impedance at tip of ear mold in ρc units,
z_e = ear impedance in ρc units (after W. Geffcken),
l_1 = length of mold (1.85 cm),
l_2 = distance from the tip of mold to ear drum (1.50 cm),
A_c = area of ear canal (0.333 cm),
A_m = area of mold canal (0.1964 cm),
A_f = cross-sectional area of main branch of filter (0.1257 cm),
A_s = cross-sectional area of neck of receiver (0.01368 cm).

The calculation of the insertion loss, in the system under consideration, due to the finite filter may be divided into three parts: first, the derivation of the formula giving the insertion loss as a function of the system impedances; secondly, an evaluation of each of the impedances; and, lastly, the numerical calculation of the insertion loss.

The principles involved in this calculation are part of standard acoustical theory and are set forth by Mason[4] and Morse.[5] We have used the "hyperbolic tangent" method of Morse but have expanded the function by the hyperbolic addition theorem obtaining the impedance at any position in the tube as a non-transcendental function of the impedance at any other place in the tube.

Derivation of Insertion Loss Formula

The two systems shown in Fig. 4 are idealized representations of an actual ear canal with an ear mold inserted in it and terminated by a normal ear drum. In one case the receiver is connected directly to the ear mold and in the other case an acoustic filter is inserted between the receiver and the mold. Both the ear canal and the mold canal are considered as straight rigid tubes

[4] W. P. Mason, Bell Sys. Tech. J. **6**, 258 (1927).
[5] P. M. Morse, *Vibration and Sound* (McGraw Hill Book Company, Inc., New York, 1936), p. 198.

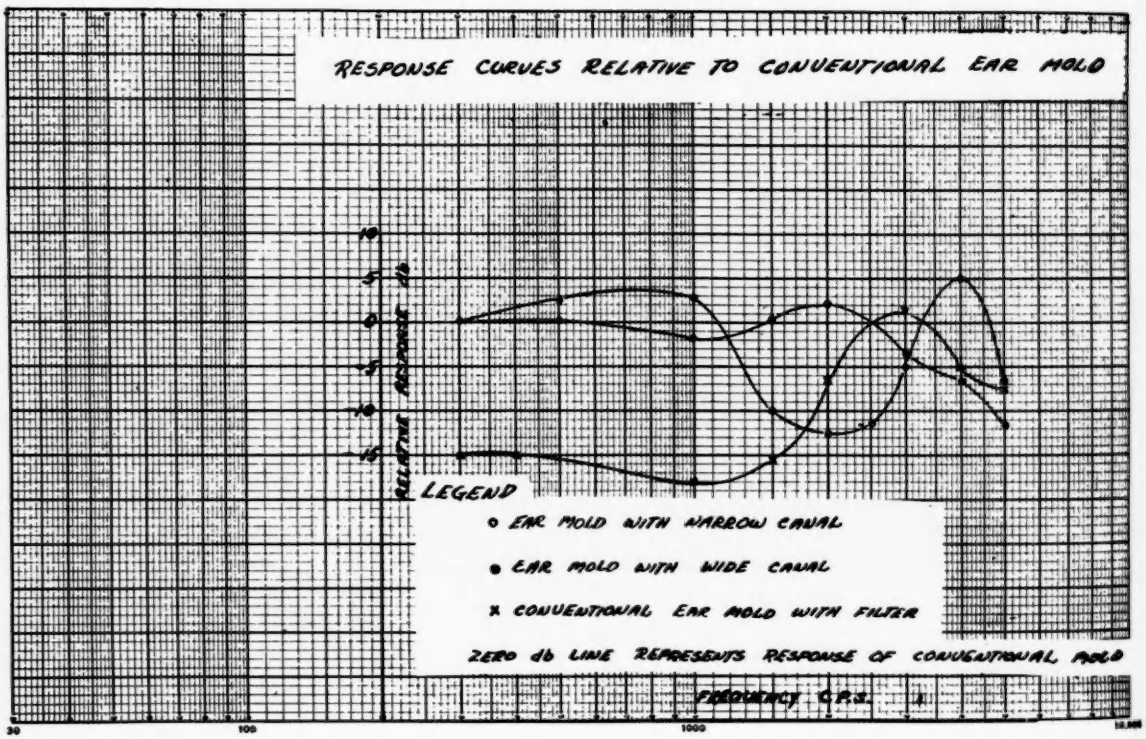


FIG. 3.

of dimensions shown in Fig. 4. In this analysis no account is taken of viscous damping of air in the mold or ear canal. Because of the width of mold and ear canal assumed, the viscous damping would probably have little effect.

The insertion loss is defined as

$$\delta = \text{insertion loss (db)} = 10 \log_{10} \left| \frac{\text{Power transmitted to the ear drum, filter absent}}{\text{Power transmitted to the ear drum, filter present}} \right|. \quad (1)$$

It is sufficient to compare the power passing into the mold at the entrance of the mold for the two cases, filter present and filter absent.

Note: Subscripts refer to position in the system as shown in Fig. 4.

Power transmitted to the ear drum, filter absent $= \frac{1}{2} r_3 \rho c |\bar{v}_3|^2 A_m$. (2)

Power transmitted to the ear drum, filter present $= \frac{1}{2} r_3 \rho c |v_3|^2 A_m$. (3)

$$\therefore \quad \delta = 10 \log_{10} \frac{|\bar{v}_3|^2}{|v_3|^2}. \quad (4)$$

We now evaluate $\bar{v}_3$ and v_3. By Thevenin's theorem

$$v_5 = \frac{p_0/\rho c}{[z_5 + (A_f/A_s) z_s]}. \quad (5)$$

Assuming the pressure inside the whole filter is constant

$$p = v_5 z_5 (\rho c) = \frac{(p_0 z_5)}{z_5 + (A_f/A_s) z_s}$$

$$= \text{pressure inside filter}, \quad (6)$$

$$v_3 = \frac{p/\rho c}{z_3} = \frac{p_0/\rho c}{z_3[1 + (A_f/A_s)(z_s/z_5)]}. \quad (7)$$

Again by Thevenin's theorem

$$\bar{v}_3 = \frac{p_0/\rho c}{z_3 + (A_m/A_s) z_s}. \quad (8)$$

Putting (7) and (8) in (4)

$$\delta = 10 \log_{10} \frac{|1 + (A_f/A_s)(z_s/z_5)|^2}{|1 + (A_m/A_s)(z_s/z_3)|^2}. \quad (9)$$

Before proceeding to the calculation of the pertinent impedances we give a brief discussion of Eq. (9). If the source impedance z_s is zero, i.e., it is a constant pressure source, the insertion loss is zero. If, however, the source impedance approaches infinity which is tantamount to a constant velocity source, we have

$$\delta = 10 \log_{10} \left(\frac{A_f}{A_m} \right)^2 + 10 \log_{10} \frac{|z_3|^2}{|z_5|^2}. \quad (10)$$

Also it should be pointed out that it is possible for δ to become negative. One case of this type occurs when z_s, z_3, and z_5 are all pure reactances and z_s and z_3 negative and z_5 positive. A glance

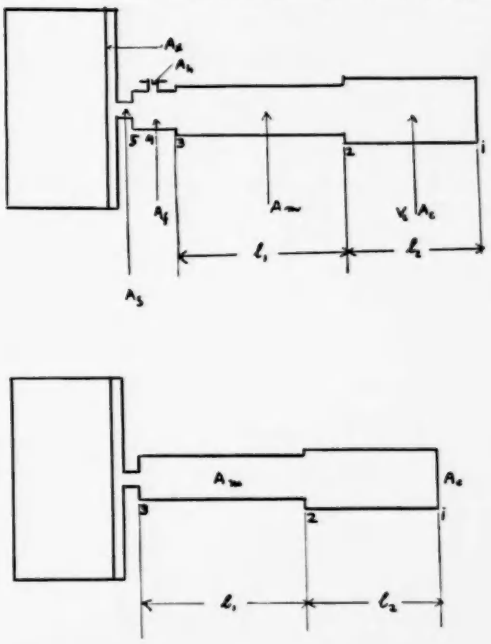
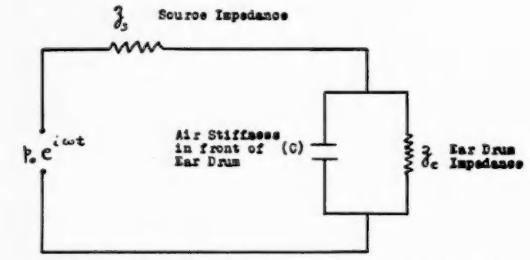
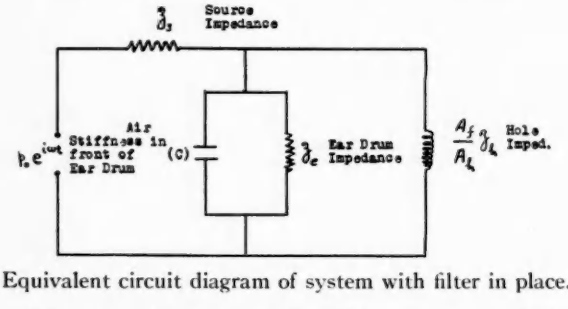


FIG. 4. Top drawing: acoustic system comprising hearing aid receiver, high pass filter, wide ear mold, ear canal, and ear drum. Bottom drawing: acoustic system comprising receiver, wide ear mold, ear canal, and ear drum. $A_d = 3.871$ cm²; $A_h = 0.0113$ cm²; $A_c = 0.333$ cm²; $A_m = 0.1964$ cm²; $A_f = 0.1257$ cm²; $A_s = 0.01368$ cm²; $l_1 = 1.85$ cm (mes.); $l_2 = 1.50$ cm (est.); $V_c = 0.50$ cm³.

at the equivalent electrical circuits will show physically how this can happen.



Equivalent circuit diagram of system without filter.



Equivalent circuit diagram of system with filter in place.

The introduction of the filter corresponds to placing an inductance in parallel with the rest of the circuit. The effect of this is to lower the impedance in the circuit containing z_s and z_h so that greater current flows in this circuit. This means that the condenser z_s must handle a large quantity of electricity and hence develop relatively large voltages across its plates. This means that the voltage across the tube-ear drum circuit will be larger when the filter is present than when it is absent. Hence under these conditions more power flows through the tube-ear drum circuit with the filter in place than when it is absent.

Of course after the source has passed, its own resonant frequency z_s becomes positive and must be represented as an inductance instead of a capacitance. For this case δ is positive. This is the more usual case encountered.

Calculation of Impedances

In order to evaluate the impedances in Eq. (9) several intermediate impedances must be computed. We now give the formulas for computing them.

z_e = Measured values of ear drum impedance. (In our subsequent calculations we have used the data of W. Geffcken.[6])

z_2 = Impedance at junction of mold and ear canal. Impedance taken on mold side looking towards the ear drum.

$$z_2=\left(\frac{A_m}{A_c}\right)\left[\frac{i\tan(\omega l_2/c)+z_e}{1+iz_e\tan(\omega l_2/c)}\right]. \tag{11}$$

This is an expanded form of the "hyperbolic tangent" formula of Morse.[5]

z_3 = Impedance at entrance to the mold, taken on mold side looking towards the ear drum.

$$z_3=\frac{i\tan(\omega l_1/c)+z_2}{1+iz_2\tan(\omega l_1/c)}. \tag{12}$$

z_4 = Impedance inside the filter, in the main tube to the right of the filter hole, looking towards the ear drum.

$$z_4=\frac{A_f}{A_m}z_3. \tag{13}$$

z_h = This impedance was calculated by assuming that the air in the hole acted as a piston in an infinite baffle in series with the mass of air in the hole. A similar formula is given by Morse.[7] Radiation was assumed from both sides of the piston.

$$z_h=2\left[\left(1-\frac{2J_1(w)}{w}\right)+\left(\frac{2}{\omega}S_1(w)+\frac{l_h}{2a}w\right)i\right], \tag{14}$$

where $w=\frac{2\pi a}{c}\nu$, a = hole radius, l_h = length of hole, $J_1(w)$ = Bessel function, $S_1(w)$ = Struve function.

z_5 = Impedance in the filter to the left of the filter hole calculated by parallel combination of z_h and z_4.

$$\frac{A_f}{z_5}=\frac{A_h}{z_h}+\frac{A_f}{z_4},$$

$$\therefore\quad z_5=\frac{1}{1/z_4+(A_h/A_f)1/z_h}. \tag{15}$$

z_s = source impedance at the junction between receiver and mold and receiver and filter, respectively, looking toward the receiver. This quantity should be determined experimentally. Since no such data were available z_s was approximated by the following method. The receiver

[6] W. Geffcken, Ann. d. Physik **19**, 829 (1934).

[7] P. M. Morse, reference 5, page 202.

TABLE I.

Frequency	z_e		z_h	z_s	z_2		z_3		z_4		z_5		δ for calculated z_s	δ for $z_s=\infty$
	r_e	x_e	x_h	x_s	r_2	x_2	r_3	x_3	r_4	x_4	r_5	x_5		
300	4.80	7.70	14.77×10^{-3}	-4.27×10^{-2}	1.007	−2.990	0.594	−2.261	0.381	−1.448	0.0069	0.186	−0.99	20.0
400	3.52	5.48	19.69×10^{-3}	-2.16×10^{-2}	0.749	−2.133	0.459	−1.594	0.293	−1.020	0.0179	0.271	−12.81	13.2
600	2.76	3.04	29.53×10^{-3}	0.56×10^{-2}	0.676	−1.332	0.429	−0.931	0.275	−0.596	0.167	0.500	1.40	3.85
800	2.31	2.99	39.38×10^{-3}	2.53×10^{-2}	0.470	−1.124	0.291	−0.675	0.186	−0.472	1.000	0.333	5.49	−5.07
1000	2.99	2.29	49.22×10^{-3}	4.18×10^{-2}	0.558	−1.006	0.335	−0.532	0.215	−0.340	0.714	−0.143	5.60	−3.28
1200	2.14	0.53	60.0×10^{-3}	6.00×10^{-2}	0.769	−0.550	0.556	−0.246	0.356	−0.157	0.419	0.0968	2.38	1.06
1400	1.71	0.50	70.0×10^{-3}	7.3×10^{-2}	0.610	−0.398	0.498	−0.034	0.319	−0.022	0.278	0.0926	1.76	2.70
1600	1.54	0.98	80.0×10^{-3}	8.5×10^{-2}	0.418	−0.413	0.350	0.080	0.224	0.051	0.197	0.115	0.65	1.81
1800	1.91	1.26	88.0×10^{-3}	10.0×10^{-2}	0.375	−0.484	0.300	0.101	0.192	0.064	0.165	0.088	0.86	2.64
2000	1.28	1.96	98.44×10^{-3}	10.75×10^{-2}	1.595	−1.367	0.435	0.142	0.278	0.218	0.225	0.142	1.14	2.78
2200	1.48	0.73	107.0×10^{-3}	12.20×10^{-2}	0.325	−0.368	0.319	0.341	0.204	0.218	0.147	0.203	1.40	3.48

was treated as a Helmholtz resonator with a non-rigid back. The neck of the receiver corresponded to the neck of the resonator. The volume between the diaphragm and the front of the receiver case was the resonator volume. The diaphragm impedance, including the air stiffness behind the diaphragm, was taken as the terminating impedance of the resonator. These considerations lead to

$$z_s=\left(l_n+\frac{16b}{3\pi}\right)\frac{\omega}{c}i+\frac{1}{\dfrac{1}{(A_sc/V\omega i)}+\dfrac{1}{(A_s/A_d)z_d}},$$

where l_n = length of receiver neck,
b = radius of receiver neck,
A_s = cross-sectional area of receiver neck,
V = resonator volume,
A_d = diaphragm area,
z_d = diaphragm impedance including the air stiffness behind the diaphragm.

In this formula no account has been taken of the resistive component of the impedance developed due to turbulence in the receiver neck nor to the resistance introduced by the cloth which some manufacturers place across the neck. Both may add appreciably to the source impedance. The calculation of the neglected components of the source impedance is very uncertain and would have complicated the problem unduly. See Table I for numerical calculations.

II. Analysis of Effect of Mold Diameter on Sound Transmission

Symbols for Analysis II

P_c = Power emanating from tip of conventional mold,
P_w = power emanating from tip of wide mold,

$$\delta=10\log_{10}\frac{\text{power received at ear drum, wide mold in place,}}{\text{power received at ear drum, conventional mold in place}}$$

v_5 = velocity of air in neck of source at entrance to expansion chamber in cm/sec.,
z_5 = impedance in neck of receiver at junction between receiver and expansion chamber looking towards the drum in ρc units,
v_4 = velocity at entrance of conventional mold in cm/sec.,
z_4 = impedance at entrance of conventional mold in units,
v_3 = velocity in conventional mold canal at tip of ear mold,
l_1 = length of conventional mold canal (2.2 cm),
z_c = characteristic impedance of conventional mold canal $=\dfrac{\Gamma_c c}{i\omega}$ in ρc units,
z_w = characteristic impedance of wide mold canal $=\dfrac{\Gamma_w c}{i\omega}$ in ρc units,
v_4 = velocity in wide mold at junction between mold and receiver in cm/sec.,
z_4 = impedance in wide mold at junction between mold and receiver in ρc units,
A_{mw} = cross-sectional area of wide mold canal (0.1964 cm),
l_1' = length of wide mold (1.85 cm),

$$\Gamma_c=\frac{1}{2c}\cdot\frac{L_c}{A_m}\left[\frac{(\gamma')^2\omega}{2\rho}\right]^{\frac{1}{2}}+\frac{i\omega}{c}\left[1+\frac{L_c}{A_m}\left(\frac{(\gamma')^2}{2\omega\rho}\right)^{\frac{1}{2}}\right],$$

Γ_w = same formula as Γ_c, only L_c and A_m replaced by L_w and A_{mw},
L_c = tube perimeter (cm) conventional mold,
A_m = tube cross-sectional area of conventional mold,

$$\gamma'=\sqrt{\mu}+\sqrt{\kappa}\left[\sqrt{\gamma}-\frac{1}{\sqrt{\gamma}}\right],$$

μ = coefficient of viscosity (c.g.s. units),
$\gamma=c_p/c_v$,
κ = coefficient of heat conductivity.

The analysis is concerned with two systems (Fig. 5). The first system consists of a hearing aid receiver coupled to a conventional ear mold which is inserted in the ear canal. It is commercial practice to place a small expansion chamber at the junction of the mold and the

receiver. Hence we have included it in our analysis. The second system consists of a mold with a wide diameter of the mold canal, connected to hearing aid receiver and inserted in the ear canal. We shall calculate the ratio of the acoustic powers received at the ear drum for each of these systems and we define the insertion loss due to the conventional mold as

$$\delta = 10 \log_{10} \frac{\text{Power received at ear drum, wide mold in place}}{\text{Power received at ear drum, conventional mold in place}}. \tag{17}$$

It is assumed that no attenuation takes place in the ear canal hence in order to calculate (δ) it is sufficient to calculate the acoustic power radiated from the tips of each of the ear molds.

Note: Subscripts refer to position in the system.

$$P_c = \tfrac{1}{2}\rho c A_m \cdot r_3 |v_3|^2, \tag{18}$$

$$P_w = \tfrac{1}{2}\rho c A_{mw} \cdot \bar{r}_3 |\bar{v}_3|^2; \tag{19}$$

also

$$r_3 = \frac{A_m}{A_c} r_2 \tag{20}$$

and

$$\bar{r}_3 = \frac{A_{mw}}{A_c} r_2.$$

Hence

$$\delta = 10 \log_{10} \frac{A_{mw}^2 |\bar{v}_3|^2}{A_m^2 |v_3|^2} = 20 \log_{10} \frac{A_{mw}}{A_m} + 10 \log_{10} \frac{|\bar{v}_3|^2}{|v_3|^2}. \tag{21}$$

Calculation of v_3 in Conventional Mold System

The theory necessary for the calculation of $\bar{v}_3$ and v_3 is given by Mason and Morse.

By Thevenin's theorem

$$v_5 = \frac{p_0/\rho c}{z_s + z_5}. \tag{22}$$

Assuming that the pressure (p) throughout the volume of the expansion chamber is constant we have

$$p = v_5 z_5 \rho c = \frac{p_0 z_5}{z_5 + z_s}, \tag{23}$$

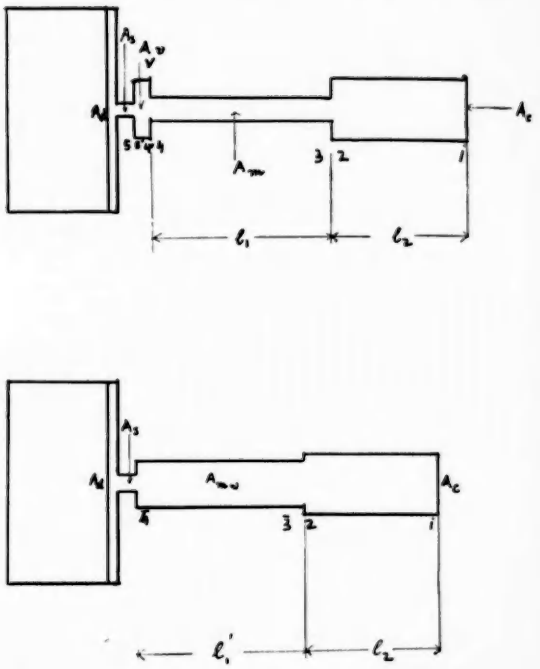
$$v_4 = \frac{p}{z_4 \rho c} = \frac{p_0 z_5/\rho c}{z_4(z_5 + z_s)}, \tag{24}$$

$$v_3 = v_4 \left[\cosh \Gamma_c l_1 - \frac{z_4}{z_c} \sinh \Gamma_c l_1 \right] = \frac{p_0 z_5/\rho c}{z_4(z_5 + z_3)} \left[\cosh \Gamma_c l_1 - \frac{z_4}{z_c} \sinh \Gamma_c l_1 \right]. \tag{25}$$

Calculation of $\bar{v}_3$ and $\bar{v}_4$ in system with mold with wide canal:

$$\bar{v}_4 = \frac{p_0/\rho c}{\bar{z}_4 + (A_{mw}/A_s) z_s}. \tag{26}$$

If we now assume that no dissipation occurs in



and this is inserted in the insertion loss formula (21). Impedance formulae necessary for the calculation of insertion loss are:

z_2, calculated previously from ear drum impedances,

$$z_3=\frac{A_m}{A_c}z_2, \tag{29}$$

$$z_4=z_c\left[\frac{z_3+\tanh \Gamma_c l_1}{1+z_3 \tanh \Gamma_c l_1}\right], \tag{30}$$

$$z_5=\frac{\left(\frac{2\pi}{c}\cdot\frac{A_s}{A_v^2}V\cdot\nu\right)i+\left(\frac{A_s}{A_m}\right)z_4}{1+i\left[\left(\frac{2\pi}{c}\right)\left(\frac{V}{A_m}\right)\nu\right]z_4}. \tag{31}$$

$\bar{z}_4$ is identical with (z_3) in analysis of filter, Eq. (12) computed neglecting dissipation in the mold. If it is desired to account for dissipation, use formula

$$\bar{z}_4=z_w\left[\frac{\bar{z}_3+\tanh \Gamma_w l_1'}{1+\bar{z}_3 \tanh \Gamma_w l_1'}\right], \tag{32}$$

$$\bar{z}_3=\frac{A_{mw}}{A_c}z_2. \tag{33}$$

CONCLUSION

Changes discussed in the acoustic line between hearing aid receiver and ear drum produced, under certain circumstances, large variations in the acoustic power transmitted to the ear drum. These variations are in agreement with previous observations and use has been made of these effects in the clinical art of fitting hearing aids.

The Baffle Effect of the Human Body on the Response of a Hearing Aid

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(Received June 5, 1944)

IT is known[1] that when a hearing aid is worn by a person its over-all frequency response is not the same as that measured when the aid is placed in a free sound field, because the human body acts as a baffle. This difference becomes of considerable importance when we realize that *hearing aids are made to be worn by people*, and that hence the response *on the person* is the valid one. This study is one of a group on the subject of putting hearing aid fitting on a more scientific basis.

The pressure at the microphone of the aid will differ from that which exists in the free field by an amount which depends on: (1) the frequency, (2) the shape and direction of the sound wave, (3) the size and shape of the person, (4) the position of the aid on the person, and (5) the clothes the person is wearing. The shape and size of the aid itself also play a part in determining the difference in its over-all frequency response when measured with and without a baffle.

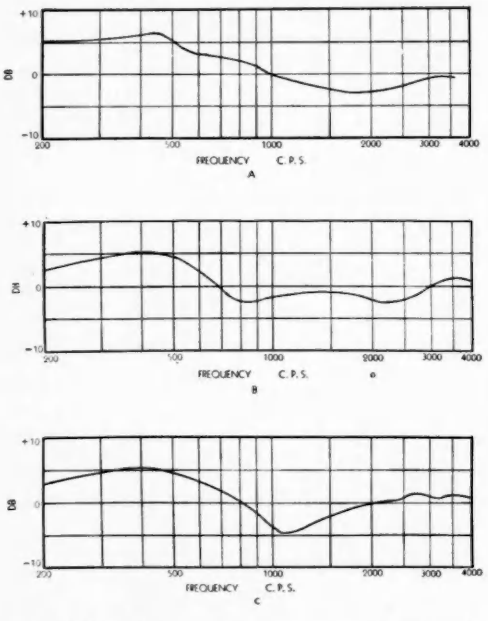

FIG. 1. Baffle effects produced by men of different sizes. *A*, large person; *B*, medium sized person; *C*, small person. Aid in center chest position.

With a view to establishing the magnitude of some of these factors, experiments were performed with several different people and with different aids. The experiments were performed out of doors to avoid room reflections. The aid was supported on a microphone stand at a distance of 7 feet from the loudspeaker (whose center was 5 feet above the ground). In all cases the aid faced the speaker. The response curves of the aid were taken with an automatic curve tracer which drove the dial of a beat-frequency oscillator at such a rate that it swept the audio-frequency band in about 1 minute. Curves were taken with and without the person standing against the aid and the differences were plotted as functions of the frequency.

EFFECT OF BODY SIZE

Figure 1 shows curves taken with an aid whose dimensions were $5\frac{1}{2}\times2\frac{3}{4}\times\frac{1}{2}$ inches. It was very nearly rectangular in shape. In all these cases the aid was at the center of the person's chest, and the persons wore similar clothing—shirt, tie, and business suit with no vest. The heights and weights of the men used in these tests are shown in tabular form below:

Curve	Height	Weight
A	5′ 11″	225 lb.
B	5′ $6\frac{1}{2}$″	150 lb.
C	5′ 2″	130 lb.

While all of these curves show that about the same over-all change in the response of the aid is made by the human baffle, the details of the curves are rather markedly different. In particular, the point at which the curves first cross the axis do not show a progressive change which can be correlated with the heights or weights of the persons used.

EFFECT OF POSITION ON BODY

To test the effect of changing the position of the aid on the body the experiments were re-

[1] F. F. Romanow, J. Acous. Soc. Am. **13**, 294 (1942).

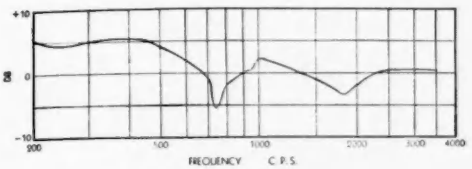

FIG. 2. Large person. Aid in left breast-pocket position.

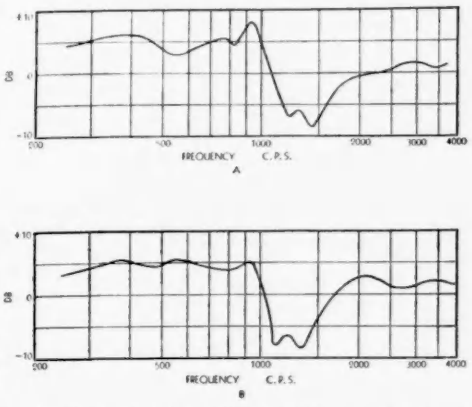

FIG. 3. Effect of using different aid. *A*, large person; *B*, same aid on small person.

peated with the persons described but with the aid in the left breast pocket position. No significant changes in the baffle effect were noted with the two smaller persons, but with the larger person the change is rather marked as shown in Fig. 2. This is to be compared with curve *A* in Fig. 1. The total magnitude of the effect, however, remains at about 10 db.

EFFECT OF DISTANCE

To determine the effect that possible ground reflections and the shape of the sound wave played in the results obtained, the aid was moved to within 4 feet 4 inches of the center of the speaker, as contrasted to the previous distance of 7 feet, and the center chest experiment repeated with the largest person. No significant change was noted in the baffle effect for this case. Experiments made previously with a cathode-ray curve tracer and with the same speaker indicated that the effect of distance was negligible down to 3 feet.

EFFECT OF TRANSMITTER SIZE AND SHAPE

To find the effect of aids of different size and shape on the baffle effect curve, an aid whose over-all dimensions were approximately $4\frac{1}{2}\times2\frac{1}{2}\times1$ inches was used. This aid was not as simple in shape as that previously described, being only approximately rectangular and more rounded at the edges. Figure 3 shows curves obtained with the largest and the smallest person, the aid being in the center chest position. Curve *A* applies to the largest person. The similarity in these curves is greater than in the corresponding curves (Fig. 1, *A* and *C*) taken with the other aid. Note the rather marked difference in the shapes of the curves taken with the two different aids under otherwise identical conditions. It is doubtful that the difference in dimensions of the aids would account for the entire change in the shapes of the curves noted.

TRIAL OF A RECTANGULAR WOODEN BAFFLE

To test the effect of simple dimensions on the baffle-effect curve a simple wooden rectangular baffle was used in conjunction with the first aid mentioned. The experimental results obtained were compared with those calculated theoretically on the basis of simple dimensions. The dimensions of the baffle were $\frac{1}{2}''\times19\frac{1}{2}''\times26''$. The aid was placed with the center of its microphone 6 inches from the top of the baffle and midway from side to side. (That is, the center of the microphone was $9\frac{3}{4}$ inches from each vertical edge of the baffle.) The experimental results obtained with this arrangement are shown in Fig. 4, *A*.

CALCULATED EFFECT

In calculating the baffle effect corresponding to this simple situation the method of virtual vibrations used by Muller, Black, and Davis[2]

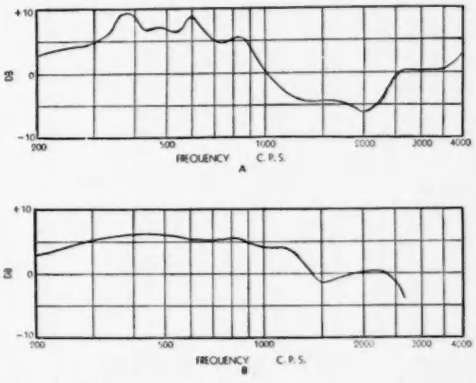

FIG. 4. Effect of artificial baffle. *A*, experimental results; *B*, theoretical results.

[2] G. G. Muller, R. Black, and T. E. Davis, J. Acous. Soc. Am. **10**, 6–13 (1938).

was employed. This method, which rests on an assumption made to avoid the difficulty in satisfying the boundary conditions of the general solution of the wave equation, assumes that each point of the surface of a baffle on which a sound wave is impinging is given a velocity equal and opposite to that of the normal component of the free space particle velocity at that point. Once these velocities are known, the total velocity potential, and hence the pressure change over the free field value at any point of the baffle, can be calculated. It was necessary to extend the results shown by Muller, Black, and Davis so that they could be applied to points on a rectangular baffle. This involved the graphical integration of the integral

$$\int_0^{\theta_1} \cos\,(2\pi D/\lambda \sec\theta)d\theta$$

up to values of $\theta_1 = 70°$ and D/λ up to values of 4.

The baffle effect of the aid itself in producing changes in free-field pressure at the center of

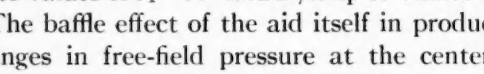

FIG. 5. Effect of clothing on artificial baffle.

the microphone was calculated and subtracted from pressure changes produced by the wooden baffle, the change, of course, being expressed in decibels. The results are shown in Fig. 4, *B*. Comparison of curves *A* and *B* shows that the baffle effect cannot be predicted by consideration of simple dimensions of baffle and aid alone. It is, of course, implicit in the theoretical calculation that the diaphragm of the microphone is flush with the surface of the baffle, but actually it is a fraction of an inch in front. In this connection it was noted that a slight change in pressure between the aid and a person acting as a baffle was sufficient to change the response several decibels, particularly at or near the minima of the baffle effect curves.

The effect of clothing on the surface of the wooden baffle was tested by clothing it with a shirt, necktie, and a man's suit coat. The curve resulting is shown in Fig. 5 and is to be compared with curve *A* in Fig. 4. The change produced is probably due not only to the change in nature of the reflecting surface of the baffle but also to an effective change in shape produced by the clothing. It is evident that the simple baffle chosen does not give an adequate representation of the effect produced by any of the human baffles used.

To test the effect of the thickness of a baffle on normally incident waves, one side of a box was used, first with the remainder of the box attached, and then without. No significant change in the resulting curves was obtained. This result is supported by the agreement between the theoretical and experimental curves of Muller, Black, and Davis for waves normally incident on circular and square baffles. Their results show, however, that their agreement between theoretical and experimental results become worse as the angle of incidence increases, particularly for angles greater than 90°. This shows that for angles of incidence other than normal, the effect of the sides of the baffle must be considered.

COMMENT AND CONCLUSIONS

It has been shown that the effective response of a hearing aid is changed approximately 10 decibels when it is worn by a person facing a source of sound in nearly free-field conditions. In general, the lower frequencies are enhanced about 5 decibels and the higher frequencies reduced, or not greatly enhanced. While this effect is large enough to warrant its consideration in specifying the frequency response of a hearing aid, its variability, even under the relatively simple condition of a normally incident sound wave, is at present sufficient to defy adoption of a single standard correction curve.

The effect which applies to one aid cannot be estimated accurately from measurements on another aid of similar dimensions even though all other conditions are assumed to be identical.

More extensive investigation of the problem is necessary to determine, first, whether or not reasonably valid average correction curves can be established for particular aids used under varied conditions and then, assuming that such averages have been established, to find an artificial baffle which will yield the average result pertaining to the particular aid with which it is used.

Pressure and Field Response of the Ear in Hearing Aid Performance Determination

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(Received April 5, 1944)

Since hearing aid receiver acoustical output is commonly measured by an artificial ear wherein a microphone replaces the eardrum, the measurement yields data only on nominal pressure at the eardrum. A correction factor must be applied to the artificial ear output to relate it to the free field performance of the ear (which is the norm). In other words, substituting pressure operation for free field operation of the human ear changes its frequency response, and this change must be allowed for in determining the corrective characteristics of a hearing aid. The variation is computed and is then compared with subjective test results. The sum of this correction and one for the obstacle effect of the human body wearing the hearing aid transmitter are then determined and it is concluded that such an over-all correction s so great that it cannot be neglected in determining or comparing hearing aid characteriistics. The over-all correction is then applied to representative instrument characteristics.

INTRODUCTION

THIS paper is one result of a group of studies of the problem of fitting electronic hearing aids to people. In the industry it has been common practice to measure the characteristics of a hearing aid transmitter with its earphone feeding a 2-cc closed coupler artificial ear. The transmitter is operated under free field conditions. Our organization some years ago observed that instruments when fitted to a person did not behave exactly as *uncorrected* measurements and audiogram indicated. At first a correction curve was determined subjectively. Later a series of studies was begun, and it was found that the fault lay with neither free field curve nor audiogram—but lay instead with conditions of use. Applying these corrections has validated direct application of audiograms as an element of fitting procedure.

THE HUMAN ELEMENT AND THE HEARING AID

There are two major changes in the conditions of use when a hearing aid is donned by a wearer (as compared to the free field test mentioned above):

A. The transmitter case is pressed up against and forms part of a much larger body. This will change the response characteristics of the microphone. An accompanying paper by Dr. Wilbur W. Hansen analyzes this in rather greater detail than was used in the only prior study.

B. An ear listening to direct speech operates under free field conditions, whereas an ear wearing a telephone receiver is pressure operated. Our measurements of a hearing aid receiver with a 2-cc artificial ear indicate performance when feeding an eardrum through a chamber and canal similar to that of the ear.

However, the free field and the pressure response of the human ear differ materially, so the mere wearing of a receiver changes listening characteristics in a readily determined manner. If the response curve of the hearing aid is to be used for fitting purposes, this change in characteristics must be incorporated in the fitting data, lest error be introduced. This paper discusses the magnitude of such error, and the sum of this ear error and the obstacle error previously mentioned.

PRIOR ART

That there is a difference between artificial ear output readings and subjective effect was recognized by Romanow.[1] Subjective methods were used by him and no basis was given for the "2-cc coupler field correction," save a general reference to Sivian and White.[2] Different curves were given for different types of receivers.

Our organization has, similarly, used a subjective correction curve for some years.

Kranz and Rudiger[3] stated that in the fitting of aids it was good practice to provide more

[1] F. F. Romanow, J. Acous. Soc. Am. **13**, 294 (1942).
[2] L. J. Sivian and S. D. White, J. Acous. Soc. Am. **4**, 288–320 (1933).
[3] F. W. Kranz and C. E. Rudiger, J. Acous. Soc. Am. **13**, 365 (1942).

high frequency and less low frequency reponse than called for by the audiogram. No data on the extent of this correction were given.

The difference between pressure response and field response of the ear was recognized and discussed by Sivian and White,[2] but no difference values were computed nor was the matter explored further.

METHOD

All data have been secured and will be presented in terms of relative response, that is, response as compared to the 1000-cycle value. This avoids the question of absolute accuracy of measurements performed by different groups at different times, and the philosophical question of what force to give the change from binaural to monaural listening. I say philosophical, because when using a hearing aid the unaided ear does hear some direct sound and we do not have pure monaural listening in a large proportion of the cases. Fortunately the difference in *relative* response due to monaural rather than binaural listening may be disregarded.

Working in terms of relative response has the further advantage that the data are more significant. It is the variation in frequency response which determines tone quality and accuracy of fitting of a hearing aid, whereas variation of 1000-cycle absolute gain may be compensated for, within limits, by a touch on the volume control.

Studies made are based on a comparison of equal loudness curves for free field at 0° azimuth and pressure at the eardrum as determined by Fletcher and Munson,[4] and Sivian and White.[2]

The method of computation is not involved. In the case of Fletcher and Munson data, for example, let

A = db free field sound pressure at 1000 cycles for a loudness of L phons,

B = db free field sound pressure at F cycles for a loudness of L phons,

C = db sound pressure at eardrum at 1000 cycles for a loudness of L phons,

D = db sound pressure at eardrum at F cycles for a loudness of L phons.

Then at loudness L:

$$\frac{\text{Pressure relative-response at frequency } F}{\text{Free field relative-response at frequency } F} = (D-B)-(C-A).$$

EFFECT OF INTENSITY

Perhaps the most important question is the relation of this effect to intensity level. Since the effect is caused by head and ear configuration one would not expect intensity to influence the matter to any degree. Figure 1 shows computed

[4] H. Fletcher and W. A. Munson, J. Acous. Soc. Am. 5, 82–108 (1933).

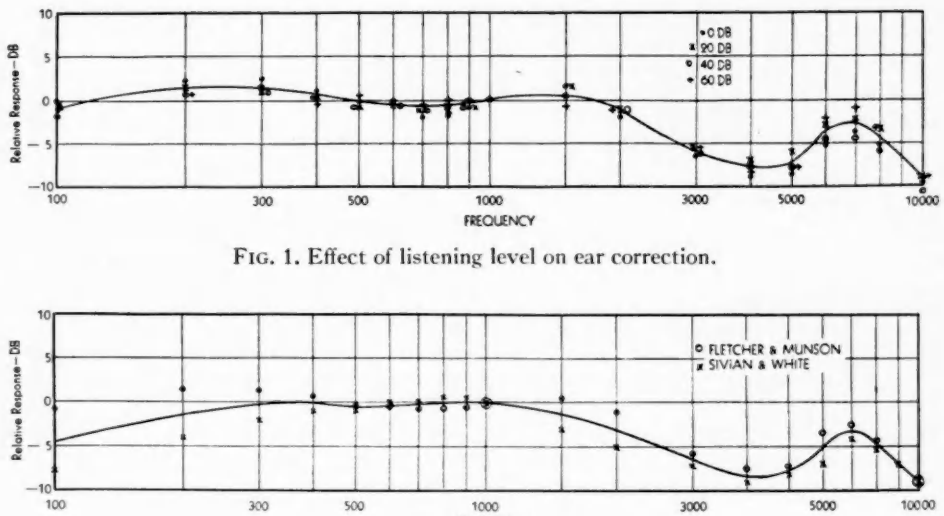

FIG. 1. Effect of listening level on ear correction.

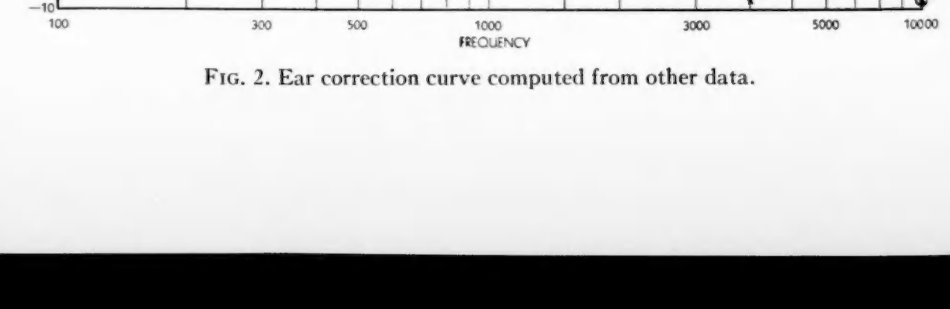

FIG. 2. Ear correction curve computed from other data.

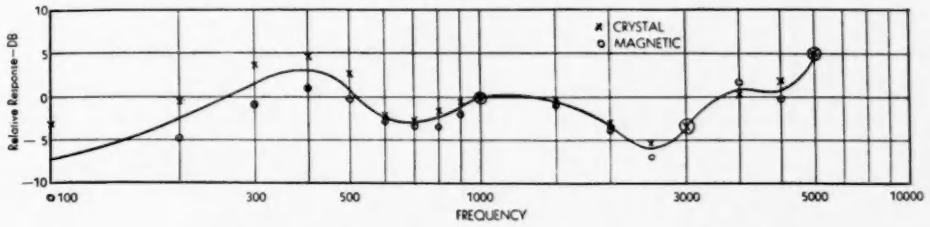

FIG. 3. Subjective observations—field calibration of 2-cc coupler.

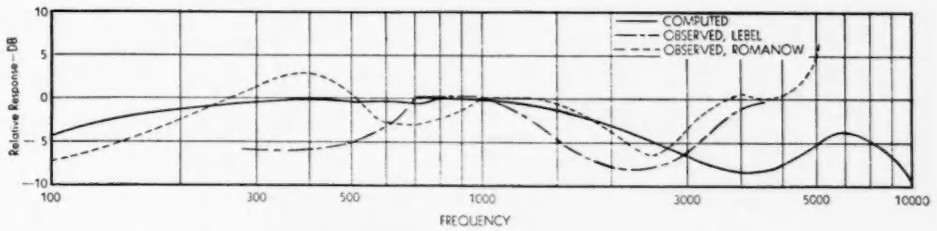

FIG. 4. Correlation of computed and observed results.

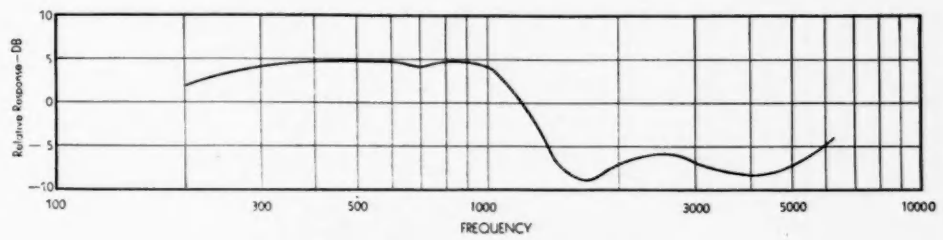

FIG. 5. Over-all hearing aid correction curve.

points taken at 0, 20, 40, and 60 db above threshold loudness, using Fletcher and Munson data.[4] The curve drawn represents the average of the 20-, 40-, and 60-db loudness levels. It may be seen that all points, regardless of level, are within 1 db of the curve up to 5000 cycles, and within $1\frac{1}{2}$ db up to 10,000 cycles. It is evident that the question of listening level may be safely neglected as a practical matter.

The curve so determined has been checked against data computed from a Sivian and White study[2] made at threshold. Figure 2 shows both sets of points and a curve giving the average. It can be seen that the average agrees with either set of points within two db from 250 to 10,000 cycles.

SUBJECTIVE MEASUREMENTS

For comparison's sake two subjective observations of 2-cc coupler field correction given by Romanow[1] have been replotted on a relative response basis and an average curve drawn in Fig. 3. The general correlation of the two sets of observations shows that we have a definite trend, plus differences due to earphone characteristics. It is interesting to note that the only major divergence occurs where the effect of imperfect seal to the ear has its greatest effect.

For further comparison the average curves previously derived have been combined in Fig. 4. It is evident that there is substantial agreement (within 2 db) from 175 to 3000 cycles, and excellent agreement (within 1 db) from 900 to 2800 cycles. Below 250 cycles the leaky seal to the ear has progressively increasing effect; above 3000 cycles the closed coupler is evidently not a sufficiently good acoustic image of the human ear.

We have made subjective tests of the field correction of a 2-cc coupler, with a result which is also given in Fig. 4. A limited number of observers and a crystal receiver were used. It is

clear that there is a qualitative agreement between Romanow's and our results. The difference below 700 cycles may be charged to a difference in the perfection of the seal to the ear canal, or to greater sensitivity to effect of seal, of the earphone used. An average of Romanow's and our observed results is within 2 db of the computed from 500 to 3000 cycles.

The field correction of a 2-cc coupler then consists of two factors:

A. The pressure *vs*. free field response of the ear.

B. Differences between acoustic impedance of the artificial and the human ear.

Factor *A* is of considerable practical importance and appears adequate correction for hearing aid purposes, especially since it seems to be very close to the over-all coupler correction curve from 500 to 3000 cycles. It is suggested therefore that the computed curve of Fig. 4 be so used.

Factor *B* should be of importance in connection with wide range and other commercial type receivers. It is suggested that the conventional coupler be modified to include a leak of fixed magnitude, to improve the low frequency accuracy. Further modification will be desirable if the coupler is to be used at frequencies above 3000 cycles. Of course the difficulty lies not so much in simple modification as in creating a reproduceable device.

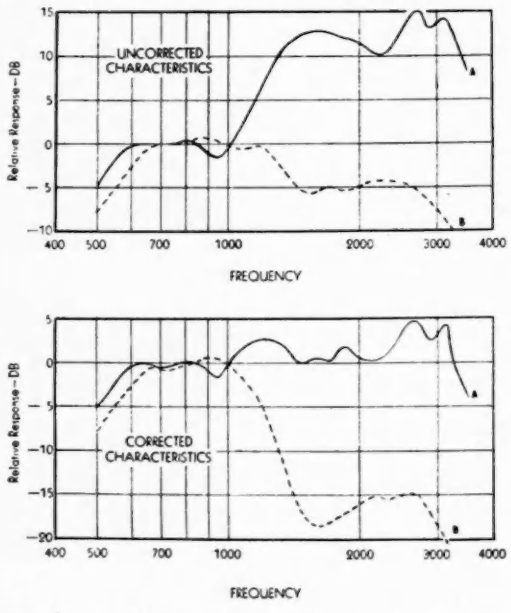

FIG. 6. Application of over-all correction curve to representative instruments.

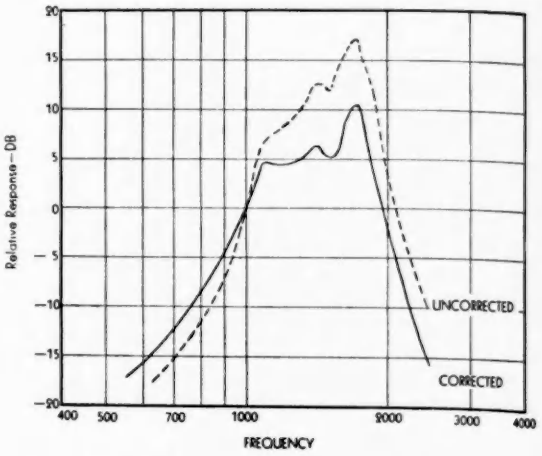

FIG. 7. Effect of correction curve on instrument of limited frequency range.

THE OVER-ALL CORRECTION CURVE

For better perspective on the obstructive effect of the body[5] as combined with the computed ear effect just discussed, the total of the two has been plotted in Fig. 5 using a typical value for the body effect. Representing an over-all shift of 13 db, it is evident that the effect must be allowed for in preparing response curves to be used for comparison or prescriptive fitting. We have been using a curve of this general nature for some time, with uniformly improved results in fitting.

To show the effect of the over-all correction curve of Fig. 5 still further, it has been applied to two commercial instruments of similar size and shape. In Fig. 6 we have corrected and uncorrected data on two units. Both are adjusted as the manufacturers advise for listeners with a uniform hearing loss. On the uncorrected graph we see that *A seems* to have rather too much high frequency response, while *B looks* much better. However, when we take the corrected relative response curves we find that the situation is completely reversed. *A* has a very uniform actual response while *B* will be very muffled in sound due to severe lack of high frequencies. Subjective tests with experienced observers correlate well with the corrected pair of curves.

This still further illustrates the fallacy of using

[5] W. W. Hansen, 1944, unpublished.

uncorrected curves "because they are at least comparative." Both sets of curves in Fig. 6 are comparative, but the ordinary observer's conclusions from one and from the other are decidedly different.

It is interesting to apply this type of correction to an instrument of much narrower frequency range. In Fig. 7 we have shown corrected and uncorrected relative response data on a very widely advertised, very low priced vacuum-tube hearing aid. As in the previous graphs, the uncorrected curve was secured as the ratio of free field input to the aid to acoustical output to a 2-cc close coupler artificial ear. The corrected curve was obtained by applying the data of Fig. 5, modified to allow for the larger case size. The hearing aid tone switch was set in the position of greatest high frequency response; in spite of that it is evident that the unit does not have enough high frequency response to correct for normal non-uniform hearing impairment. As a matter of fact, with a useful frequency range of about 900 to 2000 cycles, we have a type of response characteristic of the carbon aids of twenty years ago, yet it is evident that by virtue of its position in the tone spectrum, the use of correction data for field conditions is as important for this aid of bad quality as for a good one.

CONCLUSION

It has been shown that the corrective response of a hearing aid may be correlated with the results of normally applied measuring procedure by the use of appropriate correction factors.

Anatomical Changes Responsible for Blast Deafness and the Prevention of Such Damage to the Ear*

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(Received June 15, 1944)

THE physical energies of loud sounds and of destructive detonation, or blast, waves are of different orders of magnitude. A device that protects the ear from damage by intense sounds may, therefore, fail to prevent injury by a blast wave.

Increased pressure does not, of itself, injure the ear. This fact is proven daily by caisson workers and divers, who are often submitted to pressures equivalent to three or more atmospheres.

The rate of change in pressure is the important factor in injury of the ear by blast. At the so-called shock front of an advancing blast wave the pressure rise is extremely rapid, and the peak pressure that caisson workers sustain without injury, when gradually applied, may be reached in a time interval so brief that the advancing front of a blast wave can properly be compared to the first half of the positive pressure phase of a sound wave in the audible range of frequencies. It is no wonder that injury occurs when a pressure change of this rapidity and magnitude strikes a structure capable of detecting changes of pressure of this rapidity when the energy is of the order of magnitude of 10^{-15} watt. The overloading is too great for the delicate apparatus to withstand.

The irregular fluctuations of the phase of decreasing positive pressure, the rather prolonged negative pressure phase, and the succeeding train of intense sound waves may all cause injury to the ear or add to the injury, but it is not necessary to assume they do so in order to account for the lesions found in ears damaged by a blast wave. The single pulse of increasing positive pressure by itself offers a sufficient explanation of the observed injuries, both to the middle ear and to the inner ear.

The injury to the middle ear that may be caused by a blast wave consists primarily of tearing, or rupture, of the tympanic membrane; the injury to the inner ear consists of destruction of part of the organ of Corti, the end-organ of the cochlea. The important injury, so far as permanent impairment of hearing is concerned, is that to the organ of Corti.

The tympanic membrane is only about 0.1 mm in thickness and 9 to 10 mm in diameter. That part of the middle ear directly opposite the tympanic membrane is only from 1 to 3 mm deep; therefore, if the contained air were confined to this region it would, by "back-pressure," cushion the thin membrane before it became stretched enough to be torn. The total air capacity of the entire middle ear complex, however, amounts to several cubic centimeters, and the narrow space nearest the tympanic membrane is in free communication with the much larger pneumatized spaces of the mastoid and other regions of the temporal bone. The trapped air therefore exerts but a slight damping effect, and it is surprising that the tympanic membrane is not torn more often by blast waves than it is.

Except in rare instances the tympanic membrane, if torn, soon heals; and unless the ossicular chain has been broken, a very rare occurrence indeed, the healed middle ear is essentially normal, both structurally and functionally, provided of course that it was normal before the injury occurred and that a purulent otitis media does not develop. Infection of the middle ear seldom occurs after tearing of the tympanic membrane by a blast wave if the ear is left strictly alone; misguided efforts directed toward cleaning the external canal are responsible for most cases of otitis media in these patients. Bacteria on the skin of the external canal are washed into the middle ear through the opening in the tympanic membrane. These infected ears

* From the Otological Research Laboratory, The Johns Hopkins Medical School. Presented, by invitation, in the Symposium on the Correction and Prevention of Hearing Impairment, at the meeting of the Acoustical Society of America, in New York, May 12–13, 1944.

usually clear up soon, if promptly and properly treated, and complications seldom occur.

The injury to the organ of Corti is primarily to the hair cells, on which the nerve fibers of the auditory nerve terminate, and the damage is permanent. There is no evidence that regeneration of hair cells ever occurs, or that the highly specialized cells which support the hair cells and hold them in position can regenerate. The end result of a blast injury to the organ of Corti is therefore either a low ridge (low in comparison to the normal end organ) of irregularly shaped non-specialized cells or a single layer of flat epithelial cells resting directly on the basilar membrane. The basilar membrane itself is not ruptured by blast waves or by other forms of acoustic trauma, but a basilar membrane by itself cannot convert the energy of sound waves into nerve impulses. For the present discussion it does not matter which, if any, of the numerous theories about the functioning of the organ of Corti is correct; all agree that the presence of hair cells is essential to hearing.

Blast waves do not, fortunately for their victims, cause destruction of the entire length of the organ of Corti. Usually the total destruction is limited to one turn of the cochlea, or less; the other turns remain essentially normal or have destruction of only part of their hair cells. The functional result is, of course, that blast waves cause impaired hearing but not total deafness. The impairment for the tones perceived may be of severe degree, but the more common result, in civilian life at least, is good or fairly good hearing for some tones (usually the low ones) and extreme impairment or total deafness for other tones (usually the higher ones). Rather often an annoying tinnitus persists; this may, theoretically, be caused by chronic irritation of the nerve endings in a region of damaged, but not destroyed, organ of Corti.

Clinical observations made before and during the first World War gave the impression that rupture of the tympanic membrane by a blast wave tends to protect the inner ear. Animal experiments made by the writer in 1917 and 1918, to test the effectiveness of protective devices, gave a similar impression; the cochlear lesions usually were more severe and extensive if the tympanic membrane remained intact than if it was torn by the blast wave.*

"A finger tip placed firmly over the end of the external auditory canal sufficiently reduces the intensity of the waves that reach the tympanic membrane, and this simplest method of all would be quite satisfactory if one could always anticipate the need and had no other use for either hand. It is a method frequently resorted to by artillery men at target practice.

"Solid obturators of all types, if they fit the external canal properly, sufficiently reduce the intensity of sound and of detonation waves, but long continued wearing of them causes most persons marked discomfort, their use favors the development of infections of the external canal, etc. In addition, solid obturators are entirely unsuitable for use by those whose duties require that they be able to hear ordinary sounds well at all times. The latter objection applies also to the use of cotton or of 'waste' soaked with oil or with vaseline, both of which effectively protect an ear from acoustic trauma.

"With dry cotton tightly packed in the external canal ordinary sounds, such as commands, warning cries, etc., can be fairly well heard, but dry cotton is very ineffective in stopping even moderately intense detonation waves. This fact has been repeatedly stated in the literature on the subject, but nevertheless dry cotton is still the most commonly used method of trying to protect the hearing.

"Water-soaked cotton soon dries out enough to have air-containing spaces and loses whatever effectiveness it may at first have in protecting the ear.

"Obturators made of wax mixtures that can be shaped at body temperature to fit the auditory canal closely are effective in stopping injurious

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* To avoid the risk of including, even unintentionally, ideas or material that should not be discussed openly at the present time, in the Symposium the topic of the prevention of acoustic trauma was, for the most part, presented by extensive quotations from a paper the writer published in March, 1941, in the *Annals of Otology, Rhinology, and Laryngology*, Vol. 50. Part of the quotations are given below. That paper was itself merely a review, written at request for a medical group; therefore, those readers interested in a more comprehensive account and in illustrations of the lesions of the organ of Corti caused by blast are referred to the series of four papers published by the writer in the *Journal of Laboratory and Clinical Medicine* in 1917–1919, Vols. 2–4.

intensities of detonations and are more comfortable to wear than are solid obturators, but with them ordinary sounds cannot be heard well enough for most military duties.

"Many devices designed to permit good hearing of ordinary sounds and at the same time to stop detonation waves have been made. Some of them have been patented and produced commercially. In general, these divices utilize one or the other of three principles: (1) the elastic properties of rubber, (2) a thin diaphragm backed up by perforated metal disks, with the assembly mounted in a hard rubber or vulcanite tube shaped to fit into the external auditory canal, (3) a quick acting valve mechanism mounted like the preceding.

"All devices based on either of the latter two principles have, in common with solid obturators, an objection for military use that has not yet been mentioned. If struck by a projectile which might otherwise inflict only a minor superficial wound, the device is shattered and the fragments may cause serious wounds. Some of the devices perform rather well under laboratory conditions, but under field conditions it would be impossible to keep them clean enough to be of any value."

The older observations and principles still are valid, and the writer wishes to emphasize some of them that often are overlooked, and that at the same time support the statement made early in this paper; namely, that an ear plug may be satisfactory for protection from noise and fail to protect from blast waves.

To protect an ear from an injuriously intense blast wave, any device that can be placed in the external auditory canal must fit so snugly that the blow struck by the blast wave does not drive it inward. This means, from the practical standpoint of the wearer's comfort, that the device must be in contact with a considerable surface area of the external canal, that the contact surface must include the entire circumference of the canal wall, and that the pressure of the contact must be of moderate degree. If the pressure is more than a moderate one, the discomfort of wearing the device is too much to be long endured. Furthermore, the contact and the pressure must be constantly maintained during all the changes of shape that the external auditory canal undergoes during jaw and neck movements. Devices that depend upon the contacts made by flanges do not meet these requirements, and in the writer's experience no such device protected against blast waves severe enough regularly to injure an unprotected ear.

The ability of any device to protect the ear from blast waves can only be determined by testing with injuriously intense blast waves; the efficiency of a device cannot be evaluated by extrapolation of data for sound waves or even by extrapolation of data obtained by the use of low intensity blast waves.

The Rehabilitation Service for the Hard of Hearing at Deshon General Hospital, Butler, Pennsylvania

EDWARD H. TRUEX, JR.
Acting Chief of Service, Deshon General Hospital, Butler, Pennsylvania
(Received June 4, 1944)

1. INTRODUCTION

APPROXIMATELY one year ago it became evident to the Surgeon General of the United States Army that the deafened soldier would need special measures of treatment to fit him for a return to duty or for the economic and social competition of civilian life. The program for this therapy had its inception at Walter Reed General Hospital in Washington, D. C., but within a few months the far-reaching scope of its possibilities and the volume of patient-traffic that would have to be borne necessitated its transfer to an institution where expansion would not be limited. Deshon General Hospital in Butler, Pennsylvania was designated along with Borden General Hospital in Chickasha, Oklahoma and Hoff General Hospital in Santa Barbara, California. The project at Walter Reed was transferred to Deshon on November 1, 1943.

2. VOLUME OF PATIENTS ANTICIPATED

An attempt was made to arrive at an approximate figure which would represent the number of aural casualties requiring rehabilitative measures. Estimates by recognized authorities in the otological field varied from 50,000 to 250,000 cases as a direct result of military service. The methods of testing hearing acuity that have been used to determine a man's fitness for military service have done little more than screen out the gross hearing impairments. Hence, the Army will be obliged to rehabilitate a very large percentage of the hypacusic young adult male population whose deficiency has no actual connection with the service. At the present time our government is making compensation payments to more than 12,000 veterans of World War I for total or partial deafness.[1] Without factual knowledge, I believe it is safe to say that a great many of these hearing cases were not service connected although they are recorded as such. If one bears this figure in mind and considers that the number in the armed forces is many times as great as during the last war, it is not unreasonable to contemplate that a quarter of a million men will eventually draw pensions because of deafness. If the present size of our army is maintained for two more years and the promised invasion materializes I am sure that we can expect that at least 10,000 cases will be filtered through to the three hospitals acting as centers for deafness. Moreover, the number will increase directly with the intensity and duration of the conflagration. For every case wherein the auditory acuity is impaired to the point of deafness there are many others whose conductive or perceptive apparatus has been irreparably damaged without causing subjective or objective embarrassment. Each one will be a potential case for rehabilitative measures and during the next decade or two they will swell the war disability statistics to a considerable degree.

3. PATIENT TRAFFIC TO DATE

During and after the last war very few defective hearing patients were given beneficial training which was due in part to a lack of general knowledge that a center for such benefits existed. It became apparent when our present program was in its infancy that many handicapped soldiers were being discharged from the Army because of hearing acuity below minimal army standards; others who needed treatment were maintained in service on a limited duty status, and still others were sent back to duty merely because there was "nothing better to do with them." Immediately after the memorandum was issued from the Surgeon General's Office designating the centers of deafness the influx of patients was accelerated, but reports that came to us through devious channels indicated that many medical installations failed to heed the directive. Consequently, War Department Cir-

[1] From the Office of Budget Officer and Chief of Statistics, Veteran's Administration, Washington, D. C.

cular No. 81 was issued throughout the Army dated 23 Feb. 1944. Paragraph IV, Section 2 reads as follows:

> Every case in which the impairment of hearing shows a true loss in the better ear of 30 decibels of hearing within the conversational range (256–2048 db) . . . , in a case of stationary or progressive deafness free from acute inflammatory aural disease, will be transferred in accordance with existing regulations to a hospital designated for the rehabilitation of the deaf at the earliest practicable date.

This is an order, and its effects are being felt by the hospital staff. Our daily census of patients is rising rapidly being recorded as 168 on the 20th of March and 291 on the 1st of May, representing an increase of 73.2 percent in forty days. We anticipate a plateau in the near future followed by a rapid rise to a peak load within a few months after "D" day.

Between November 1, 1943 and May 1, 1944, a period of six months, several hundred patients were admitted to the Rehabilitative Service at Deshon. During the first four months, that is, from November 1, 1943 until March 1, 1944 when the War Department order was circulated, the monthly mean of admissions was one-half of the number admitted in April of 1944. As of the 1st of May more than 100 new patients had been authorized for admission from various medical installations throughout the eastern part of the country and were merely waiting for local orders and available transportation.

4. CASES CAUSED BY ACOUSTIC TRAUMA

Of particular interest to the members of this society, I believe, is the number of casualties whose hearing was partially or totally destroyed by acoustic trauma. A complete anamnesis is taken on all patients in an effort to elicit the etiological factor or factors responsible for the underlying pathology. Since we can place very little reliability on aural observations as recorded on the induction charts, we are obliged to accept the patient's word that his ears were normal before the injury occurred. Only 14.7 percent of cases to date were recorded as having hearing loss which resulted solely from the trauma of explosive concussion or noise. An additional 4.8 percent had evident or professed disease of the ears before exposure to the violence that caused further serious deterioration of sound reception. Thus approximately one out of five cases resulted from injury. We are not permitted as yet to give any additional analytical data concerning the causes or characteristics of the abnormalities encountered.

5. PERSONNEL

The original directive from the Surgeon General had stated that research was to be encouraged. Having this in mind and being aware of the numerical possibilities of the material for study, it was natural to envision that significant contributions could be made in all phases of the hearing problem—medical, acoustical, pedagogical, and psychological.

We sought and gratefully received counsel from many of the country's leading otologists and scientists who had a primary interest in the field. Their cooperation and advice made it apparent that an opportunity was at hand to increase the existing knowledge of deafness. Some of them are now lending their capable assistance in taking advantage of that opportunity. They have facilitated the acquisition of personnel who have had the academic training and experience to analyze the problems, offer possible solutions, and assess the value of these solutions. Every person now associated with the organization was individually selected for his qualifications for a particular division of the labor at hand. In order to accomplish this we had to acquire, in addition to Army personnel, the services of 22 people on a civilian basis under Civil Service regulations. This includes 19 lip-reading teachers, 1 speech teacher, 1 acoustic physicist, and 1 teacher for training in the use of residual hearing.

a. Medical

The four otologists are officers in the Army of the United States, and none of them has any illusions about the restoration of natural hearing to the chronically deafened. Since the beginning of modern medicine all types of chronic hearing afflictions have defied treatment and although the otologist has recognized temporary defeat, he has failed to maintain his responsibility to the victims of this malady. If the staff at Deshon

can expedite a universal medical supervision of deafness and the deafened, then its contribution will be significant. It is highly impractical for a physician to assume the full care of these patients unless he intends to devote his entire time to "hearing" as a specialty. It is imperative that centers, similar to those the Army has organized, be established throughout the United States under medical control. The few such civilian centers that now exist are doing invaluable work and are operating more efficiently than the juvenile military program but they touch only a small number of the population in need of therapy.

b. Acoustical

Primarily our problem is one of fitting the individual with the most satisfactory hearing aid and teaching him the art of reading lips. The Army has authorized us to purchase any hearing aid that has been accepted by the Council of Physical Therapy of the American Medical Association for any patient regardless of the duration or cause of his affliction. Personnel who are recognized in the field of acoustics have been acquired and are now supervising the audiometric and speech reception tests, the application of newly devised tests, and the scientific dispensing of hearing aids. It is hoped that a simplified, accurate system of selecting an instrument will evolve from the studies that are now under way.

In addition, a qualified research organization has associated itself with us in order to utilize the resources of patient-material to improve existing methods of auditory measurement and to improvise new ones, the objective being to engender refinements in the construction of hearing aids, the testing of a patient, and in the system of fitting the instruments to the patients.

c. Psychological

The numerous psychological aberrations that frequently stalk in the wake of deafness have proven too complex for us who are amateurs to cope with. In this field, too, the vast source of material to be studied offers a challenge for creative work. By May 15 the position of clinical psychologist will be very adequately filled, thus eliminating the vacancies in all key positions.

d. Pedagogical

The principal of the school of lip-reading at Deshon has had twenty-five years of experience, largely devoted to the training of teachers in that field. The twenty-two instructors, all graduates of accredited normal schools, gave 3800 lessons during the month of March each lesson lasting forty-five minutes. One teacher devotes her entire time to advanced students in practice groups. Another will train the patient by special moving pictures to utilize visual and kinesthetic clues for the comprehension of an entire situation. Still another gives the patient special instruction in the use of his instrument which requires a cerebral reorientation to the auditory stimuli of every day noises. Minor abnormalities of speech have been noted so frequently in the group of patients already studied that one speech correctionist has been employed, and the necessity for another has arisen.

6. OUTLINE OF REHABILITATION

When a soldier is admitted for rehabilitation he is interviewed and examined by an otologist who makes a tentative diagnosis and institutes the necessary corrective measures not only for his auditory afflictions, but for any evident or occult physical abnormality that is revealed by clinical and laboratory study. He is tested in sound-conditioned rooms with the pure tone audiometer and the spoken voice. (I might add that *soundproof* rooms will be in use in a matter of days, and speech reception testing is gradually being instituted for diagnosis and for the comparative evaluation of personal hearing aids.) A Lucite ear mold is made, and the tedious process of giving the patient a suitable hearing aid is begun. He is enrolled in a course of instruction designed to reorient him to sounds that are transmitted by the instrument. He begins a curriculum of study in the school of lip-reading which consists of daily individual and group-practice classes of instruction. During his stay in the hospital trained social workers, members of the American Red Cross, endeavor to solve his

personal problems, usually economic or domestic, maintain the necessary contacts between the hospital and his family, and finally bridge the gap between military service and the Veteran's Facility which negotiates contacts with the civilian agencies that will continue any additional rehabilitative measures that he might need.

In approximately eight weeks the average hard-of-hearing soldier has attained sufficient proficiency in the use of his residual sensations to adapt himself satisfactorily to any situation he encounters. He is returned to limited duty or discharged from the Army depending upon the degree of his disability and the Army's need for him as a special individual.

SUMMARY

The program for rehabilitation of the hard of hearing at Deshon General Hospital was designed to treat at least one-third of the many thousands of patients whose hearing is or will be below certain standards. Selected specialized personnel is being utilized to restore these patients to the Army or to civilian life, equipped with the maximum available benefits for this particular affliction by medical, acoustical, pedagogical, and psychological methods.

Acoustical Society News

May, 1944 Meeting in New York Outstanding Success

The New York meeting of the Acoustical Society on May 12 and 13 drew the unusually large registered attendance of 340. The symposia on hearing and supersonics were especially well attended.

Robert S. Shankland
Chairman of the Program Committee of the May, 1944 meeting

The most important person in the Acoustical Society at any moment is the Chairman of the Program Committee; it is he that works the hardest; it is he that carries the burden of assembling a significant program which will be of interest and value to the members. The credit for our recent meeting goes largely to Dr. Robert S. Shankland, Chairman of the Program Committee. Other members of the Committee were Leo L. Beranek, Vern O. Knudsen, Keron C. Morrical, and L. J. Sivian. They are to be congratulated on the success of their efforts.

Biennial Award for Noteworthy Contributions to Acoustics Presented to Dr. Leo Beranek

At the banquet in the Hotel Pennsylvania during the New York meeting of May 12 and 13, the Biennial Award for Noteworthy Contributions to Acoustics was presented to Dr. Leo L. Beranek by President Floyd A. Firestone Mr. Hugh Knowles introduced Professor F. V. Hunt who read the following biographical sketch of his illustrious student.

Biographical Sketch of Leo Leroy Beranek

The career of Leo Leroy Beranek is in the best American tradition, as characterized by the expression "local boy makes good in the big city." Leo was born on September 15, 1914, in Solon, Iowa, and here, in due course, entered grammar school. He soon began to reveal to those who had the wit to understand those characteristics of independence and aggressiveness which have brought him before us tonight. A silk-hose selling venture returned an income of $5 per week, and at the age of twelve, he was working as assistant projectionist in the local motion picture theatre. At the tender age of fourteen, he moved to Mt. Vernon, Iowa, and within a year was conducting his own radio repair business with a clientele drawn from a thirty-mile circle surrounding the town.

These business ventures accumulated a comfortable financial reserve with which Leo entered Cornell College, in Mt. Vernon, in the fall of 1932. You may imagine his discomfiture when simultaneously with his entrance a local bank failed and his liquid assets were completely lost. Undismayed, Leo set to again and, starting from scratch, earned his way through Cornell College by dint of working in restaurants, a dry-cleaning establishment, as drummer in a dance band, as an undertaker's assistant, and, with a partner, in the management of his own radio store. The

Leo L. Beranek

radio store wound up with a $1000 inventory of radio sets which he and his partner closed out during the final month of college. I might add that the less said, the better, concerning the peculiar adaptability of a hearse for transporting public address equipment.

Leo graduated from Cornell College with distinction in 1936 with the Bachelor of Arts degree and was awarded a Gordon McKay Scholarship for the two succeeding years, providing for study in physics and communication engineering in the Graduate School of Engineering at Harvard University. This work led to the degree of Master of Science in Engineering in 1937. During the following two years, Leo pursued his study for the doctor's degree while he served as Research Assistant in the Department of Physics. In the spring of 1939, Harvard awarded to Leo the Parker Traveling Fellowship entitling him to a year of travel and study in any part of the world. Since the year 1939–40 was not an opportune time to study leisurely in Europe, Leo elected to remain at Harvard to continue his studies in architectural acoustics. This led to a thesis on "Precision measurements of acoustic impedance" and the degree of Doctor of Science in the spring of 1940. In September, 1941, he married Miss Phyllis Knight. From 1940 to 1943, Dr. Beranek held appointments as instructor in physics and communication engineering at Harvard University, resigning this post in 1943 in order to devote himself more completely to war research work.

Dr. Beranek's contributions to the literature of acoustics began in 1939 with a paper on "The analysis of sound decay in rectangular rooms." He has taken an active interest in the affairs of the Acoustical Society, having presented several papers at our semi-annual meetings, and having become, tomorrow, a member-elect of the Executive Council. He has translated and abstracted more than twenty-two papers from foreign periodicals for the Abstract Section of our journal and has published four additional research papers in the journal containing significant contributions to the theory of room acoustics, the technique of acoustical measurements, and the propagation of sound in ducts.

In the fall of 1940, his program of work in architectural acoustics was interrupted by the demand of government agencies for studies in military acoustics. Under Dr. Beranek's direction, a research group was formed, expanded, re-expanded, and then enlarged. This group is still actively working on a variety of military problems dealing with air-borne sound. The first of these problems to be undertaken was a study of the reduction of high noise levels in long range bombers. Not very much can be said about this work except to tell you that it was outstandingly successful and that it has reduced the problem of the acoustic treatment of airplane interiors to an exact and calculable science capable of achieving, with the lightest possible treatment, any reasonable sound levels which may be written into the specifications. Intercommunicating equipment for airplanes and other types of military vehicles has also been improved so materially as to revise completely previously accepted standards for this type of service. Other projects about which even less can be said have been known to some of the members of your Award Committee and would support similarly enthusiastic accounts of other phases of Dr. Beranek's work.

In assessing the value of the scientific research of a young man, one attempts to examine the continuity of the record in order to extrapolate the curve into the undetermined future. Such extrapolation leads to an optimistic prediction for Dr. Beranek since the quality and the quantity of the work, past, present, and continuing, upon which this award is based, constitute a record of accomplishment of which all of us may well be proud. The Acoustical Society honors itself by honoring him.

F. V. HUNT.

Northwestern Symposium on Hearing Aids and Residual Hearing

The School of Speech of Northwestern University is conducting a Symposium on Hearing Aids and Residual Hearing from June 26 to August 26. The list of lecturers includes Dr. Howard Carter, Dr. Edmund P. Fowler, Mr. Joseph B. Kelly, Dr. Scott N. Reger, Dr. S. S. Stephens, and Dr. Norman Watson. The symposium includes a series of lectures on backgrounds in speech and hearing, nature of hearing aids, industrial considerations, methods for selecting the proper hearing aid, research frontiers, and the effective use of residual hearing.

New Members

James W. Ballard
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Physicist

Thomas G. Barnes
Division of Physical War Research
Duke University
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Physicist

Dwight N. Brown
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Acoustical Contractor
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Deceased: Stuart Ballantine, Fellow
William S. Purser, Member

Resigned: Charles H. Randolph, Member

Transferred to Fellowship: Benjamin B. Bauer, Richard L. Brown, E. Dietze, George W. Downs, Louis W. Labaw, Vincent F. Mallory, L. W. Sepmeyer.

Fellows	179
Members	762
Total membership	941

Stuart Ballantine
1897–1944

Stuart Ballantine

Stuart Ballantine, prominent radio engineer and a leader in the field of electro-acoustics, died May 7, 1944, after a short illness. A Fellow of the Society, Mr. Ballantine had demonstrated exceptional technique and facility in solving acoustical problems since entering this field around 1928.

Of greatest popular appeal was his development of the throat microphone for the U. S. Army Air Corps during 1934–37, the first of its type to be tested and standardized for use in U. S. military aircraft. Picking up voice sounds directly from the larynx, the throat microphone excluded engine and propeller noise, freed the pilot's hands for more important duties, and permitted use of oxygen masks. Thousands of these have been rendering useful service since their development.

Mr. Ballantine was born September 22, 1897, in Philadelphia, Pennsylvania. At a very early age he became an adept student of radio telephony dating experience as an amateur radio operator back to 1908. After several years acquiring stature, he broadened his efforts by becoming a commercial operator for the Marconi Company during the summers of 1913–16. Following other commercial experiences he became Expert Radio Aide with the U. S. Navy from 1917–20, distinguishing himself at this time by improving the Navy coil-type radio compass.

Returning to academic pursuits after World War I, Mr. Ballantine entered the Graduate School at Harvard University, achieving election as John Tyndall Fellow in Mathematical Physics in 1923. Currently he established relations with the Radio Frequency Laboratories at Boonton, New Jersey, and commenced an extensive period of research which resulted in numerous inventions and technical manuscripts contributing greatly to the improvement in broadcast reception during the growth of this new industry.

Turning his attention to acoustics in 1928, Mr. Ballantine discovered large errors in then existing methods of calibrating condenser microphones due to diffraction and cavity resonance. He proposed the "free field" method of calibration which has supplanted the older methods and also proposed a new technique of calibration which is now replacing the older thermophones. For this and other work he was awarded the Morris Liebmann Memorial Award of the Institute of Radio Engineers in 1931. He continued the development of microphones and loudspeakers along with improvements in radio circuits to enhance fidelity of radio reception and expended much effort to establish "high fidelity."

The throat microphone development followed in 1934 and led to increasing investigations concerning communication in aircraft radio which continued up to the time of his passing. He was deeply engaged in research on technical problems relating to the war effort up to the last.

He was the author of some fifty papers on scientific subjects and in the course of this work was issued about thirty patents on various devices.

In addition to being a Fellow of the Society, Mr. Ballantine was a Fellow of the Institute of Radio Engineers, also its President in 1935; Fellow of the American Physical Society; member of the Franklin Institute and recipient of the Elliott Cresson Gold Medal from that body in 1934; member of the Radio Club of America, American Institute of Electrical Engineers, American Association for the Advancement of Science, and American Society for Metals.

Current Publications on Acoustics

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Book Reviews

Transients in Linear Systems. Volume I. By MURRAY F. GARDNER AND JOHN L. BARNES. Pp. 390. John Wiley & Sons, Inc., New York. Price $5.00.

This useful volume deals with operational calculus methods, using the Laplace transform technique, as applied to electrical and mechanical systems with lumped parameters. It is readable, but difficult for persons not familiar with the usual mathematics of a.c. circuit analysis. For persons who are familiar, it provides a satisfactory review of this recently developed alternative method of studying vibrating systems.

For all electrical and mechanical systems where storage of energy is involved, the theoretical solution for the behavior of the system can be handled in several ways. The classical method is to study the free oscillations of the system and to express all motions in terms of free oscillations and modifications of free oscillations. This method is termed, at present, the method of eigenfunctions. The natural frequencies (or eigenvalues) and the characteristic functions (or eigenfunctions) are first found, and the general motion of the system is then expressed in terms of a series of the eigenfunctions.

This technique has enjoyed considerable success both in the classical fields of acoustics and electromagnetic theory and also in the modern field of wave mechanics. Its greatest usefulness comes in studying the free vibrations of the system after an instantaneous initial impulse or displacement is given to the system. It also is fairly easy to use in studying systems where a simple harmonic driving force is applied at one point.

The method of eigenfunctions, however, becomes difficult to use when energy dissipation occurs, or when the driving force is a more complicated function than the one just mentioned. When energy dissipation occurs, in other words when there is effective resistance in the system, the eigenvalues become complex instead of real, and the eigenfunctions are often not an orthogonal, complete set. This makes it exceedingly difficult to use the method in studying the general motion of the system; and for a good many years workers have usually succumbed to the temptation of neglecting friction in working out the behavior of complex oscillatory systems.

Unfortunately, in a great many cases, the neglect of energy dissipation means a neglect of fundamentally important properties of the system. A neglect of absorption of energy by the walls of a room, for instance, would make it impossible to calculate the important acoustical properties of the room. Similarly, the study of electrical circuits without the introduction of resistance would be a rather academic pastime. In response to the need for a more general method of studying electrical circuits involving dissipation, a method has been developed which is now termed operational calculus. This method begins from the simple driven motion rather than from the free vibration. Instead of the natural frequencies we have the impedance of the system as a function of driving frequency; and instead of the various eigenfunctions we have the behavior of the system for each point application of force and for each frequency. In this method of attack resistance plays a straightforward role; in fact, its presence in many cases makes the problem easier to solve than if it were absent. To obtain the general solution, combinations of the different solutions for different frequencies are then combined. The technique for combining these solutions is what is termed operational calculus.

Operational calculus began somewhat shrouded in mystery. Its great protagonist, Heaviside, was not given to particularly clear explanations. It was not until comparatively recently that the relationship between the operational calculus methods and the methods of earlier mathematical investigators was brought out clearly. It is now apparent that the operational calculus is a particular application of the more general method of solving differential equations by means of the integral transform, sometimes called the method of Green's functions. In the case of most problems dealing with oscillations, the transform is either a specialization or a generalization of the Laplace transform.

The book of Gardner and Barnes brings out the use of the Laplace transform in a very clear-cut manner. The authors have kept the balance between reasonable clarity and reasonable mathematical rigor with commendable dexterity. The novice will certainly find the book hard sledding in spots, and the mathematician will certainly deplore the lack of rigor in spots, but it seems to the reviewer that the criticisms would be about equal in number, and that this result is the best one can expect to have in a textbook.

The examples dealt with, as might be expected, are mostly concerned with electrical systems, although there is a considerable amount of discussion of mechanical systems. The present volume concerns itself with systems composed of lumped elements, the sort of problems usually dealt with in low frequency electrical circuits, and occurring more seldom in acoustical cases. It is to be hoped that the second volume will concern itself with the more general problem which arises in transmission line theory and in room acoustics.

The first chapter takes up a comparison of linear electrical and mechanical systems, and a discussion of the type of problems which the operational calculus (or the Laplace transform method) is most useful in handling. The next

four chapters then deal with a detailed study of the Laplace transform and its application to the study of linear circuits. Various types of driving functions are considered, and the techniques of including initial conditions are dealt with. A table of Laplace transforms and their inverses is given, and details are worked out for the obtaining of these transforms for more complex expressions. In Chapter 7 there is a discussion of the complete solution of one-dimensional electrical and mechanical problems, where the most general cases are discussed and a number of examples are given. In Chapter 8 is a discussion of special properties of the Laplace transform which are useful in the solving of a number of problems.

The final chapter is a very interesting discussion of the application of the Laplace transform to linear difference equations with constant coefficients. Difference equations turn up in ladder network circuits, motions of a weighted string, and in the unconnected fields of actuarial science and switchboard practice. The methods discussed appear to have considerable practical applications, though it is doubtful whether their use in acoustics will be of great importance.

This first volume by Gardner and Barnes is not a book for the novice in mathematics; however, it is a very useful book for the person, versed in the mathematical methods of modern electrical engineering, who wishes to study the techniques of operational calculus and its possible applications to acoustics. The reviewer believes this book is rather more easy to read than most volumes on operational calculus and that it is one which well repays study.

PHILIP M. MORSE
Massachusetts Institute of Technology

Tables of Lagrangian Interpolation Coefficients. Prepared by the Mathematical Tables Project of the Work Projects Administration of the Federal Works Agency. Pp. 392. Columbia University Press, New York. Price $5.00.

The simplest type of interpolation consists in considering the function as linear in the small interval between two known values. More accurate interpolation requires consideration of the curvature of the function, the rate of change of curvature, etc.; this type of interpolation requires that values of the function be known at a number of points in the neighborhood of the interpolated value. These tables implement a simple procedure for finding the interpolated value of the function when the values of the function at a number of equally spaced values of the argument are known. The tables cover the cases of interpolation from three-point up to eleven-point data.

The procedure consists in multiplying the values of the function at each of the considered points by a different coefficient taken from the table (which coefficient depends on the fraction of the argument interval to which the interpolation extends) and summing these results. Surprising accuracy of interpolation can be achieved by this method; an example is given in which five-point interpolation of a mathematical table yields a value correct to twelve significant figures.

The introduction explains the theory of the method and gives illustrative examples. It might have been useful to provide an additional section outlining the procedure for the use of the tables for interpolation without requiring that the user become somewhat familiar with the theory.

F. A. FIRESTONE
University of Michigan

Table of the Bessel Functions $J_0(Z)$ and $J_1(Z)$ for Complex Arguments. Prepared by the Mathematical Tables Project of the Work Projects Administration of the Federal Works Agency. Pp. 403, 4 charts. Columbia University Press, New York. Price $5.00.

Bessel functions of orders zero and one are encountered in the general solution of boundary value problems in acoustics when the domain is bounded by a circle or a circular cylinder. They are used in discussing the vibrations of circular plates, diaphragms, circular membranes, or the oscillatory motion of a sphere in a viscous medium. When the influence of dissipation is considered in the aforementioned problems, the solutions involve the use of Bessel's functions of complex argument.

These tables give $J_0(Z)$ and $J_1(Z)$ with an accuracy of ten decimal places with the complex argument covering a range of 0.00 to 10.00 in intervals of 0.01 in the absolute value, and in intervals of five degrees in angle. The tables are of convenient form and interpolation procedures are carefully outlined.

F. A. FIRESTONE
University of Michigan

Table of Circular and Hyperbolic Tangents and Cotangents for Radian Arguments. Prepared by the Mathematical Tables Project of the Work Projects Administration of the Federal Works Agency. Pp. 410. Columbia University Press, New York. Price $5.00.

The main table in this volume is devoted to the circular and hyperbolic tangents and cotangents for radian arguments ranging from zero to 2.0 at intervals of 0.0001. Supplementary tables for all four functions over the range from zero to 10.0 at intervals of 0.1 are also included. These tables may be regarded as forming a companion volume to the *Tables of Circular and Hyperbolic Sines and Cosines* which appeared in 1939. The tables in this volume are given to 8 significant figures. Fifteen-place conversion tables are provided for radians to degrees and degrees to radians.

F. A. FIRESTONE
University of Michigan

Table of Reciprocals of the Integers from 100,000 through 200,009. Prepared by the Mathematical Tables Project of the Work Projects Administration of the Federal Works Agency. Pp. 201. Columbia University Press, New York. Price $4.00.

References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this journal issued in November, 1939. The same Classification of Subjects can be found in the index to Volume 15.

Compiled with the generous assistance of Samuel Sass, Physics Librarian, University of Michigan.

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2. Architectural Acoustics

2.2 **Acoustical Design and Treatment for Speech Broadcast Studios.** E. J. Content and L. Green, Jr. I.R.E. Proceedings **32**, 72–77 (Feb. 1944).

2.6 **Noise Reduction; Acoustical Materials: Their Selection and Use.** H. R. Sleeper. Arch. Rec. **95**, 101–109 (March, 1944).

2.9 **Sound-Damping of Walls Against Speech and Music.** J. Capek. Akust. Z. **7**, 152–156 (July, 1942), Sci. Abs. **A47**, 135 (1944).

2.9 **Noiseless Ductwork in a Radio Studio.** J. E. Hubel. Sheet Metal Worker **35**, 31–32 (Feb. 1944).

2.12 **Sound-Damping of Walls Against Speech and Music.** J. Capek. Akust. Z. **7**, 152–156 (July, 1942), Sci. Abs. **A47**, 135 (1944).

2.12 **Sound-Deadening Afforded by New Asbestos Composition.** Sci. Amer. **170**, 132 (March, 1944).

2.12 **The Problem of Reduction of Vibrations by Use of Materials of High Damping Capacity.** A. Gemant. J. Appl. Phys. **15**, 33–42 (Jan. 1944).

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4.4 **High Frequency Deafness and Discrimination of "High Frequency" Consonants.** R. N. Plummer. J. Speech Disorders **8**, 373–381 (1943).

4.4 **Prevention and Treatment of Acoustic Trauma.** G. E. Shambaugh. J. Speech Disorders **8**, 369–372 (1943).

4.4 **Measurement of Hearing and Deafness.** Nature **153**, 87–88 (Jan. 15, 1944).

4.4 For Papers on Deafness, see Volta Review.

4.5 **Training for Optimum Use of Hearing Aids.** S. Richard Silverman. The Laryngoscope, St. Louis (Jan. 1944).

4.5 **New Zenith Hearing Aid.** P. Wilde. Consumers Res. Bulletin **13**, 24–25 (March, 1944).

4.6 **Measurement of Hearing and Deafness.** Nature **153**, 87–88 (Jan. 15, 1944).

4.6 **Octave Generalization, Pitch Discrimination, and Loudness Thresholds in the White Rat.** H. R. Blackwell and H. Schlosberg. J. Exp. Psychol. **33**, 407–419 (1943).

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5.51 **Acoustical Design and Treatment for Speech Broadcast Studios.** E. J. Content and L. Green, Jr. I.R.E. Proceedings **32**, 72–77 (Feb. 1944).

5.7 **Standard Musical Pitch.** Science **99**, No. 2562, Suppl. p. 11–12 (Feb. 4, 1944).

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5.15 **Measurement of Extremely Small Sound Pressures with the Condenser Microphone.** G. Weymann. Elekt. Nachr.-Techn. **20**, 149–158 (June, 1943), Wireless Engineer **21**, Abs. 470 (1944).

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5.17 **Microphones and Receivers, with Special Reference to Speech Communication.** L. C. POCOCK. Elect. Comm. **21**, 218–232 (1944).

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9.1 **Polarized Speech.** L. CLOUGH. Electronics **17**, No. 4, 402–403 (April, 1944).

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10.3 **On Supersonic Waves in Cylindrical Tubes and the Theory of the Acoustic Interferometer.** P. E. KRASNOOSHKIN. Phys. Rev. **65**, 190–195 (March 1/15, 1944).

10.3 **Applications of Supersonics in Metallurgy** (two paragraphs). J. Sci. Instrum. **20**, 200 (Dec. 1943).

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THE JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA VOLUME 16, NUMBER 1 JULY, 1944

Review of Acoustical Patents

ROBERT W. YOUNG
C. G. Conn, Ltd., Elkhart, Indiana

Printed copies of patents are furnished by the Patent Office at a cost of 10¢ each. Address all communications to: The Commissioner of Patents, Washington, D. C. Remittance may be made by money order, certified check, cash, or special coupons which are sold in blocks of twenty or books of one hundred.

The decimal numbers appearing before titles are those which identify the headings being used in the analytic subject index of this journal. The Patent Office classification given in parentheses with each patent entry is outlined in the "Manual of Classification of Patents" which is obtainable from the Superintendent of Documents for $1.50.

Unless otherwise indicated, patents listed have been issued by the United States Patent Office.

Reviewers

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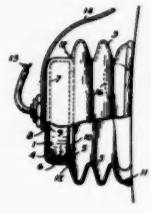
L. W. SEPMEYER, *U. S. Navy Radio and Sound Laboratory, San Diego, California*
R. W. YOUNG, *C. G. Conn, Ltd., Elkhart, Indiana*

2,336,669

4.5 EARPHONE SUPPORT AND SILENCER CUP

Ray W. Brown and Robert Iredell, assignors to The General Tire & Rubber Company.
December 14, 1943, 4 Claims (Cl. 181–22).

By this earphone cup for excluding outside noise, pressure is applied to the head instead of the ear. The

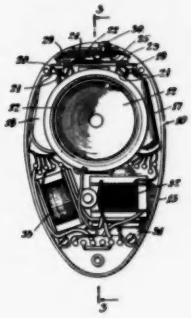


cylindrical bellows are an important feature of the invention.—LWS

2,336,828

4.5 PORTABLE MICROPHONE AND AMPLIFIER

William A. Zarth, assignor to Dictograph Products Company, Incorporated.
December 14, 1943, 2 Claims (Cl. 179–107).



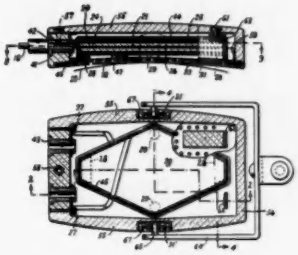
A wearable vacuum-tube hearing aid in which the components are arranged substantially as shown in the figure.—LWS

2,339,148

4.5 BONE CONDUCTION RECEIVER

Richard W. Carlisle, assignor to Sonotone Corporation.
January 11, 1944, 3 Claims (Cl. 179–107).

This patent describes a small bone conduction receiver employing a bimorph Rochelle salt crystal **21** of the bender type, resiliently mounted by pads **28** and **29** in an enclosing case; the forces are thus transmitted to the mastoid



bones by inertia reaction. Means are also described for building into the mounting a filter to attenuate the transmission of sounds above 4000 cycles.—LWS

2,339,173

4.5 BONE CONDUCTION RECEIVER

Heiman W. Koren, assignor to Sonotone Corporation.
January 11, 1944, 7 Claims (Cl. 179–107).

"Among the objects of the invention is such inconspicuously small wearable hearing aid bone conduction receiver formed essentially of a plate or strip-like piezoelectric crystal unit, at least one side of which is united, as by cementing, to a mechanically strong, elastically compliant bone-contact wall to form therewith a mechani-

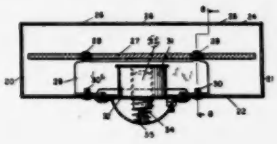
cally strong vibrating structure which vibrates under the action of electric speech-frequency oscillations applied to the crystal unit."—BBB

2,333,515

5.1 CHIME

Robert B. Brewer and Lawrence G. Bergeron, assignors of one-third to S. Eldridge Sampliner.
November 2, 1943, 5 Claims (Cl. 177–7).

The sound produced by a bar chime is said to be prolonged by mounting the bar within a resonator whose air cavity has a natural frequency of vibration equal to that of the bar. The exact manner in which the air vibrates is not explained. The bar is supposed to be nearly as wide as the cavity so as to divide it effectively into two halves.



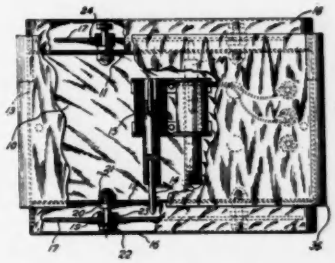
The striking mechanism is within the resonator. Openings **26** preferably have a total area one-fifth of the area of the wall in which they are situated.—RWY

2,333,875

5.1 CHIME UNIT

Carlton L. McMullen.
November 9, 1943, 3 Claims (Cl. 177–7).

The constructional details of this annunciator chime include resonators **16** surrounding the vibrating bars **17**



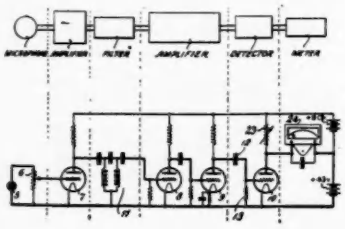
which are attached through appropriate non-metallic inserts **24** by single studs to the horizontal flanges **11**. The striking mechanism is between the resonators.—RWY

2,340,714

5.4 METHOD AND APPARATUS FOR SOUND ANALYSIS

Alfred E. Traver and Carl E. Habermann, assignors to Socony-Vacuum Oil Company, Incorporated.
February 1, 1944, 4 Claims (Cl. 73–51).

According to this patent the conventional sound level meter is inadequate to measure the motor knock in internal combustion engines. In order to overcome this difficulty the inventors use what amounts to an average peak reading metering circuit in which the time constant of the resistor and condenser combination **12** and **13** is made long in comparison to the time interval between knocks.



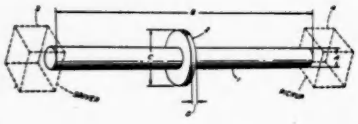
The variation of current through meter **24** is further smoothed out by a condenser across it. Normal motor noises are excluded by high pass filter **11**.—LWS

2,342,813

5.4 MECHANICAL WAVE FILTER

Warren P. Mason, assignor to Bell Telephone Laboratories, Incorporated.
February 29, 1944, 18 Claims (Cl. 178–44).

This patent teaches how mechanical band pass wave filters may be built in which pass bands are narrower than



those achieved when the central antiresonant flexurally vibrating element is a rectangular bar. The result is attained by using a central element in the form of a disk. If two points of maximum attenuation are desired, two disks are employed.—LWS

2,342,869

5.4 WAVE FILTER

Thomas G. Kinsley, assignor to Bell Telephone Laboratories, Incorporated.
February 29, 1944, 31 Claims (Cl. 178–44).

According to this patent it has been customary to place the central rectangular elements in a mechanical band pass filter near and parallel to each other. This can result in spurious tuning-fork-like vibration of these central elements thus giving rise to dips in the attenuation characteristic. To overcome this the elements are placed 90°

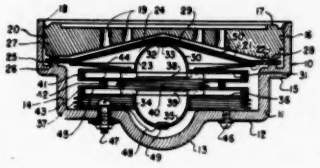
with respect to each other and preferably so that the axes intersect. Also disclosed is a method of mounting the filter by wires **10**, as illustrated in the figure for a two-section filter.—LWS

2,331,044

5.9 SOUND TRANSLATING DEVICE

Harold C. Pye, assignor to Automatic Electric Laboratories, Incorporated.
October 5, 1943, 4 Claims (Cl. 179–122).

Carbon microphone comprising two movable buttons. Button **30** is driven by diaphragm **21**. Button **35** is coupled to button **30** through a low pass filter system consisting



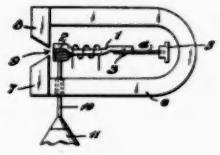
of rings and washers **31**, **36**, **41**, **42**, **43**, **44**, and **45**. Low frequency response is thereby eliminated, it is asserted.—BBB

2,338,909

5.9 ELECTROMAGNETIC OSCILLATING DEVICE WITH FREELY OSCILLATING ARMATURE

Carl Ernst Eichhorn; vested in the Alien Property Custodian.
January 11, 1944, 1 Claim (Cl. 175–339).

Electromagnetic oscillating device of the moving armature type. It is asserted that wide amplitude motion of the armature is obtained through the use of an undulatory air gap or an undulatory shaping of the armature end.



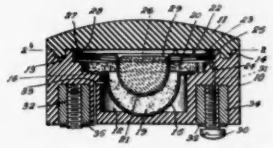
Armature is sectionalized to avoid "short circuiting" of the magnetic structure.—BBB

2,340,777

5.9 THROAT MICROPHONE

Harvey H. Stanley, assignor to Kellogg Switchboard and Supply Company.
February 1, 1944, 10 Claims (Cl. 179–126).

"This invention relates to telephone instruments in which an electric current is modified by passing through a pair of relatively movable electrodes connected by granular

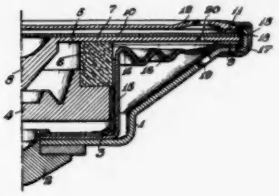


microphonic material, such as carbon, the resistance of which is variable under the influence of relative movement of the electrodes which is effected by sound vibrations." —BBB

2,345,078

5.9 CARBON MICROPHONE

Albert Ueberschuss; vested in the Alien Property Custodian.
March 28, 1944, 6 Claims (Cl. 179–122).



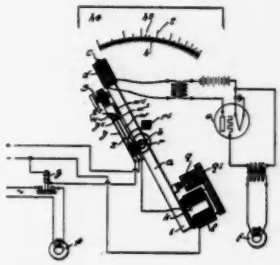
Describes the use of moistureproofing diaphragms for sealing microphone.—BBB

2,339,588

5.10 ECHO SOUNDING APPARATUS

Donald Orr Sproule, assignor of one-third to Arthur Joseph Hughes and one-third to Henry Hughes & Son Limited.
January 18, 1944, 6 Claims (Cl. 177–386).

"This invention relates to improvements in echo sounding apparatus and has for its principal object to simplify such apparatus and to reduce the weight thereof, thereby enabling such apparatus to be applied more readily to aircraft for the determination of altitude. To this end the oscillating bar **a** is provided with and carries inertia operated switches having suitably adjusted periodic times so that when the bar **a** comes into contact with the spring **q** during its return, the resilient pendulum **w** moves away from the pad w^1 and effects electrical connection through the contacts w^2 and x^2 thus establishing an electrical



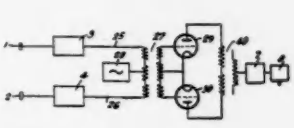
circuit . . . through . . . the winding of the timing magnet **l** . . . thereby attracting the bar **a**, bringing it against the stop **s**, and compressing the spring **q**. Further movement of the pendulum **w** in the same direction brings the contact x^2 into electrical connection with the contact y^2 thus establishing an electrical circuit to the winding **z** of the contactor for the sound pulse transmitter **p** . . . thereby causing a sound pulse to be emitted. The pendulum **w**, due to its resiliency, swings back toward the bar **a**, thereby permitting the contacts y^2 and x^2 to disengage and thereafter disengaging the contacts x^2 and w^2. . . . The magnet **l** is thus deenergized, thereby permitting the spring **q** to urge the bar **a** in a clockwise direction." The return of the echo at **f** causes magnet **d** to attract one of the silvered reeds **h** so as to indicate the elapsed time.—RWY

2,336,276

5.13 METHOD OF RECORDING AND REPRODUCING STEREOPHONIC SOUND VIBRATIONS

Marius Johannes Cuinirus van der Meulen; vested in the Alien Property Custodian.
December 7, 1943, 8 Claims (Cl. 179–100.3).

This patent describes a method of recording two-channel stereophonic sound wherein one channel is made to modulate the positive peaks of a carrier and the other channel modulates the negative peaks. The advantages claimed for this method are that only a single sound track need be



recorded and reproduced, thus requiring no complicated multi-track recording or reproducing system. The disadvantage is that the highest frequency which may be reproduced in either channel is one-half the carrier frequency. —LWS

2,336,624

5.16 PHONOGRAPH PICK-UP APPARATUS

Oran T. McIlvaine.
December 14, 1943, 10 Claims (Cl. 179–100.41).

In this phonograph reproducer of the photoelectric type vane **8** is attached to the stylus and intercepts the light

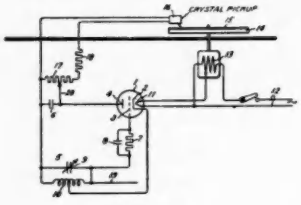
beam from lamp to photo-cell **11**. In order to increase the efficiency of the system the front surface of the vane is made reflective so that the intercepted light is reflected into photo-cell **13**. The two cells may be utilized in a push-pull amplifier.—LWS

2,336,855

5.16 ELECTRON DISCHARGE OSCILLATION GENERATOR

George W. Fyler, assignor to General Electric Company.
December 14, 1943, 5 Claims (Cl. 179–100.4).

By utilizing the Edison effect the only power required in this device for an oscillating circuit tuned to the radio broadcast band is supplied from a special tap on turntable



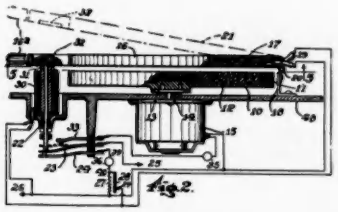
motor **13**. The oscillator is modulated by a phonograph reproducer, preferably of the crystal type. The modulated carrier radiated by antenna **19** may be picked up by a conventional radio set.—LWS

2,340,161

5.16 APPARATUS FOR OPERATING PHONOGRAPHS AND PROCESSING RECORDS THEREFOR

Harry R. Van Deventer, assignor, by mesne assignments, to Telephone Answering and Recording Corporation.
January 25, 1944, 19 Claims (Cl. 274–1).

This patent covers a "perpetual" phonograph record in that the recording is done on a "wax" surface **12** carried in turntable **11**. Above the turntable is a heater element **16** which may automatically be put into position to erase the



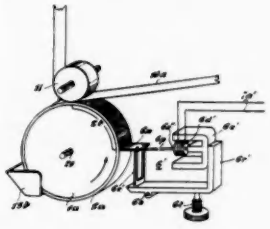
record. Means are described for controlling the heating and cooling.—LWS

2,340,562

5.16 SOUND RECORDING DEVICE

Secundino Rey.
February 1, 1944, 8 Claims (Cl. 179–100.1).

This patent discloses an improved process for direct printing of a sound track over that disclosed in Patent 2,341,652. A viscous printer's ink is applied to a smooth



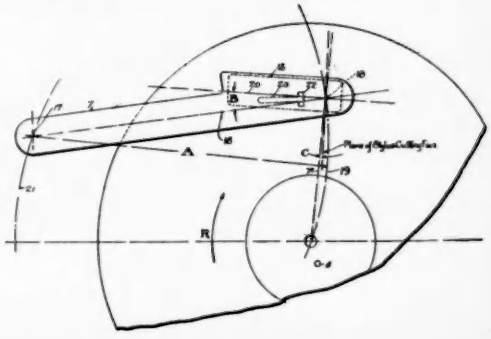
drum to a thickness of from 2 to 6 microns. The coating is then modified in accordance with sound vibrations by means of blade **6m** and then transferred to the tape **10a** by roller **11**. The process may be varied greatly in that instead of direct printing the drum may be coated with other "impression receiving materials" ranging from low melting point alloys to paraffin wax. These materials may either be inked for immediate printing or may be used for the preparation of a matrix from which the sound track is printed.—LWS

2,340,806

5.16 SOUND RECORDING APPARATUS

Willard J. Green, assignor to Philco Radio and Television Corporation.
February 1, 1944, 1 Claim (Cl. 274–1).

This patent covers a sound recorder in which the cutting



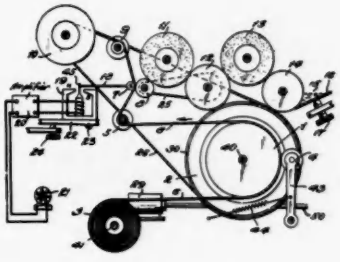
head is pivotably mounted and wherein the cutting face of the stylus is "angularly displaced in a clockwise direction from the radial line between the center of the disk and the point of cut, said angular displacement varying throughout the recording operation . . . to approximately zero at the conclusion thereof."—LWS

2,341,652

5.16 PROCESS FOR RECORDING BY DIRECT LITHOGRAPHIC PRINTING, SOUNDS IMPINGING ON MICROPHONES OR THE LIKE

Secundino Rey.
February 15, 1944, 9 Claims (Cl. 179–100.1).

A sound track of the variable density type is produced by means of lithographic printing on a narrow strip of material such as paper. In the figure the inking roller **8** receives ink from reservoir **15** through the various rollers **11, 12, 13, 14.** The pressure of the strip **6** against the printing roller **8** is varied in accordance with sound pressure by means of



roller **7** which is actuated by reproducer **19.** The initial pressure with no sound present is controlled by adjusting screw **24.**—LWS

2,342,464

5.16 COPYING SOUND EVENTS TAKEN IN AMPLITUDE RECORDS

Gustav Fries; vested in the Alien Property Custodian.
February 22, 1944, 3 Claims (Cl. 101–401.1).

By a method similar to that used in photoengraving copies of sound records are produced from a copper matrix. —LWS

2,342,679

5.16 MATERIAL FOR RECORDS

Harald Mediger; vested in the Alien Property Custodian.
February 29, 1944, 3 Claims (Cl. 274–41).

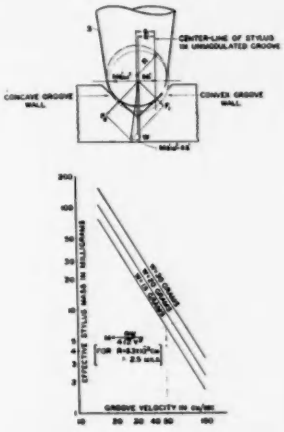
This patent covers the use as a base material, in phonograph records made in accordance with the Edison or Berliner methods, of a linear synthetic condensation material having a crystalline structure of the group consisting essentially of superpolyamides, polyureas, and polyurethanes.—LWS

2,343,417

5.16 PHONOGRAPH PICK-UP

Otto Kornei, assignor to The Brush Development Company.
March 7, 1944, 4 Claims (Cl. 274–1).

A reproducer whose stylus is designed in accordance with the dimensions given in the figures is claimed to exhibit no



frequency discrimination over a wide range.*—LWS

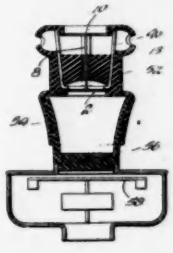
* See discussion in J. Soc. Mot. Pict. Eng. **37**, 569–590 (1941).

2,342,087

5.17 EAR SHIELD AND ACOUSTIC TRANSMITTING DEVICE

Edwin S. Pridham, assignor, by mesne assignments, to The Magnavox Company.
February 15, 1944, 5 Claims (Cl. 179–182).

This device is claimed to be an aid to communication in the presence of noise. Noise of random incidence has practically no net effect on diaphragm **10,** whereas speech sounds originating close by are said to exert a force which



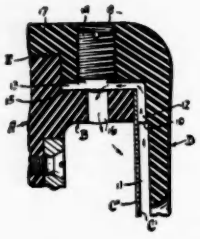
through the driving member **8** actuates the acoustically protected diaphragm **2.** This diaphragm in turn operates the telephone transmitter in the usual way. An embodiment of the device in an earcap is also disclosed.—LWS

2,342,334

5.17 VARIABLE BAFFLE FOR TRANSLATING DEVICES

Frank J. Faltico, assignor to The Maico Company Incorporated.
February 22, 1944, 8 Claims (Cl. 181–31).

In a headphone receiver or the like, the invention provides an adjustable air passageway between the front and



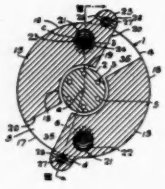
back of the diaphragm. The adjustment is made by means of screw **19** which may be brought against passage **16**. —BBB

2,317,090

5.21 TORSIONAL VIBRATION DAMPER

Paul S. Vaughn, assignor to American Locomotive Company.
December 11, 1941, 7 Claims (Cl. 74–574).

A torsional vibration dampener of the type wherein the spring factor is furnished by centrifugal force acting on a rotating mass. Web **4** is rigidly fastened to the main shaft **a**.



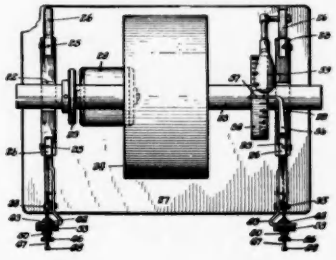
Relative motion, due to torsional vibrations in the main shaft, between parts **15** and **4** is limited by roller **24**, which is held in contact by centrifugal force tending to keep parts **15** at the outermost radial position. Any torsional vibration is opposed by the force required to move parts **15** against the action of centrifugal force.—PHG

2,328,114

5.21 VIBRATION AMPLITUDE AND CENTER OF OSCILLATION INDICATOR

Ira A. Weaver and Clyde H. Phelps, assignors to Weaver Manufacturing Company.
January 5, 1942, 5 Claims (Cl. 73–53).

A device to be used in conjunction with a dynamic balancing machine for determining the plane passing through the axis of rotation and the center of mass of a rotating body **24**. The amplitude of vibration of the body as rotated by motor **28** through belt **29** may also be obtained. The unbalance causes stroboscopic lamp **59** to flash. This occurs regularly twice each revolution when



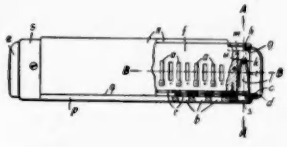
contactors controlled by screws **48** have been appropriately adjusted. The numbers shown by this illumination on the rotating scale **58** locate the center plane of oscillation. Amplitude is indicated by electrical means upon turning knob **26** by an amount sufficient to rotate scale **58**, as viewed by stroboscopic light, by one division.—PHG

2,335,453

6.1 CHROMATIC HARMONICA

Karl Scherer; vested in the Alien Property Custodian.
November 30, 1943, 4 Claims (Cl. 84–377).

The usual chromatic harmonica has a button or knob for moving the slide which uncovers another set of reeds. In



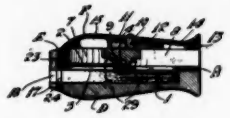
this harmonica one of the rounded ends is pivoted at **g**, so a force applied by the hand will cause the slide **c** to move lengthwise.—RWY

2,339,790

6.1 HARMONICA

Finn H. Magnus.
January 25, 1944, 16 Claims (Cl. 84–377).

For the purpose of minimizing the length of this harmonica, "blow" reed **1** is aligned with the "draw" reed **2** by being attached by the same rivet **4**. Separate "sound"



cells **7** and **8** are formed for the "draw" and "blow" reeds, respectively, but there is a common "wind" cell opening at **18** for both.—RWY

2,340,333

6.1 HARMONICA

Finn H. Magnus.
February 1, 1944, 14 Claims (Cl. 84–377).

"With this construction, it will be observed that the slots **9** constitute wind cells for the 'blow' reeds while the slots **10** constitute wind cells for the 'draw' reeds; and the slots **11** constitute sound cells for the 'blow' reeds while the slots **12** constitute sound cells for the 'draw' reeds. Normally the wind cells for the full tone 'blow' reeds are in communication with the corresponding wind openings **4** [in a plate along the lower edge], and similarly the sound cells for the full tone 'draw' reeds are in communication

with the corresponding wind openings **4**. By sliding the reed plate and cell blocks to the left wind cells for the full tone 'blow' reeds and for the full tone 'draw' reeds are closed by the portions of the wall **2** of the casing between the wind openings, and simultaneously the wind cells for the half tone 'blow' reeds and the wind cells for the half-tone 'draw' reeds are brought into communication with the corresponding wind openings."—RWY

2,341,082

6.1 KEYBOARD FOR ACCORDIONS

Ernst Claude.
February 8, 1944, 1 Claim (Cl. 84–376).

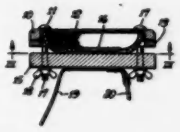
This key system for a musical instrument such as an accordion is arranged to correspond with the lines and spaces of the musical staff. For example, the buttons in

row **II** are intended for notes which are written on the lines. In between (but in row **I**) are the buttons for the notes in the spaces, and in row **III** are to be found the accidentals. It would seem that the advantage of the correspondence of key mechanism to music would be partially offset by the fact that a different fingering is required in adjacent octaves.—RWY

2,338,816

6.4 MUSICAL INSTRUCTION DEVICE

Lee M. Lockhart.
January 11, 1944, 3 Claims (Cl. 84–411).



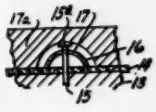
This practice drum is intended to be fastened to the thigh of the player. Tension on the drum head **12** is adjusted by tightening the wing nuts **18**. The support **16** may be made of glass.—RWY

2,339,971

6.4 XYLOPHONE

Victor Zimmerman, assignor to Harry Zimmerman.
January 25, 1944, 8 Claims (Cl. 84–403).

In this xylophone the bar **16** rests on insulating pad **14** and is located by pin **15**. The hole **17** in the bar of semicircular section is sufficiently large to permit the bar to be



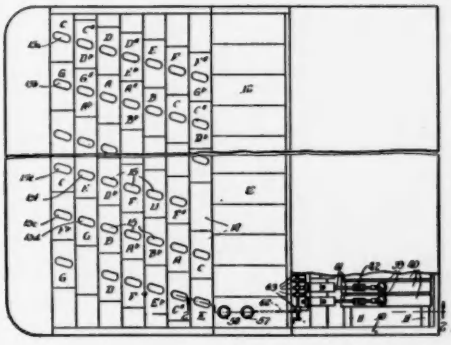
slipped over the head **15a**. The retaining rail **17a**, with its semicircular notches, then keeps all bars from being shaken from the pins.—RWY

2,341,725

6.4 MUSICAL INSTRUMENT

Charles P. Kimmel.
February 15, 1944, 18 Claims (Cl. 84–404).

Under the title of a pedal piano this percussion musical instrument is designed to be played by the feet of a dancer.



The mechanism is protected against accidental damage from the weight of the player. Since the "keys" **15** are

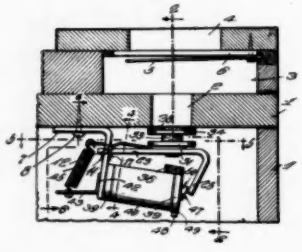
operated by relatively large forces they need to travel only small distances. A flat spring **42** in the hammer shank permits the hammer to contact bar **40** but momentarily. Two pedal boards are illustrated, one in which the keys are arranged chromatically in a series of fifths and another by major and minor thirds in each direction. In the first the diagonal keys give augmented fifth intervals whereas the second has perfect fifths along the diagonal.—RWY

2,335,311

6.5 MULTIPLE MAGNETIC VALVE FOR ORGANS

William H. Reisner, assignor to The W. H. Reisner Manufacturing Company, Incorporated.
November 30, 1943, 7 Claims (Cl. 84–339).

Here shown applied to a reed organ, this valve is designed for compact installation in a series extending along bracket **7**. Claim 2. "An electromagnetic valve actuating



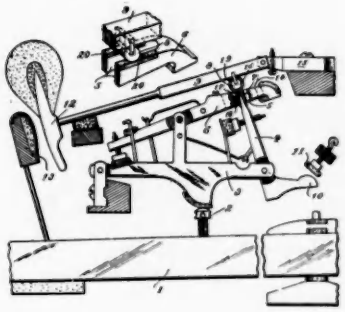
mechanism comprising a bracket member of magnetic material having a mounting portion and a supporting portion for a multiplicity of electromagnets, said supporting portion being substantially comb-like in contour and supporting a magnet core member in each of the comb-like portions of said supporting portion, armature members pivotally mounted in said bracket member and rockable with respect to an associated electromagnet, a valve carried by each armature member, and means for tensioning each of said armature members for operation by the associated electromagnets."—RWY

2,334,444

6.6 ESCAPEMENT ACTION FOR GRAND PIANOS

Reinhard Schulze.
November 16, 1943, 5 Claims (Cl. 84–239).

It is said that in the usual grand piano action the double cushioning on the knuckle **8** tends to stretch and harden under the impact of the bare wood jack, finally resulting in broken parts. "By this invention the knuckle **8** has a round rigid (uncovered) center **16** and the head **7** of the jack is cushioned by an underlay of cloth **17** and an overlay of leather **18** in such manner as to present a flat face to strike the knuckle **8**. Even should the cushion on head **7** be compressed it will not groove or pit, it will remain flat, and it will always perform its function of striking the knuckle and easily sliding therefrom when moved at right



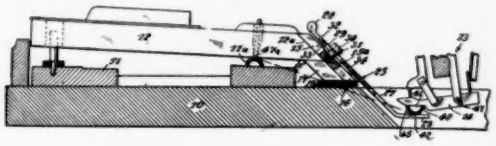
angles to the face of the knuckle **8** which was struck, because of the knuckle **10** of the jack striking its button **11**. The sensitiveness of touch will remain."—RWY

2,338,992

6.6 OPERATING MEANS FOR PIANO ACTIONS

Paul H. Bilhuber, assignor to Steinway & Sons.
January 11, 1944, 3 Claims (Cl. 84–240).

In a small upright piano it is necessary to keep the action as low as possible while at the same time the keybed must not be so low as to cramp the knees of the player. This patent discloses a method of making such a compromise, which



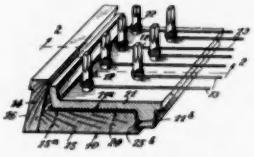
consists of a bar **25** appropriately aligned by keys **33** and **34** and capable of adjustment by screw **30**. The angle of the end **28** and the shape and location of button **45** are so chosen as to minimize the slip between these parts.—RWY

2,339,752

6.6 PIANO PIN BLOCK

Paul H. Bilhuber, assignor to Steinway & Sons.
January 25, 1944, 2 Claims (Cl. 84–188).

It has been found that the effectiveness of the pin block of a piano in retaining the pins in a given position is improved when the wood is dry. "The improved process consists in applying to the upper surface of the pinblock a



layer of resin paste in plastic state, impressing the lower surface of the string frame upon said layer, subjecting the impressed layer to the action of high frequency electrical

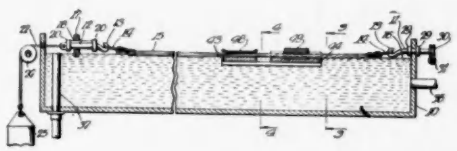
heating to harden the same and congeal the impressions, subjecting the pinblock to the action of high frequency current to remove the moisture from the body of the pinblock, and enclosing the pinblock by an envelope of water repellant material to prevent the entrance of moisture in the pinblock."—RWY

2,334,170

6.7 PROCESS AND APPARATUS FOR MAKING GUT STRINGS

Carl B. Bjorkman.
November 16, 1943, 25 Claims (Cl. 57–35).

In this apparatus for making gut strings the skein of gut ribbons is floated on the surface of water. The spindle **11** is



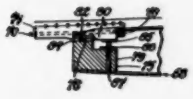
rotated by power transmitted through worm **31**. Spindle **12** is kept from turning by key **18**, but it is free to move longitudinally as the skein shortens while being wound into a "string." The tendency of an accidentally long ribbon to sag is overcome by the flotation and also by the guide block **43**. This is important in order to promote uniformity when the string is later finished to size by sanding. The molding plates **44** and **49** may be carried along to the left under power at such a rate that the gut to the right is twisted some 75 turns to the inch. Since several sets of spindles are all operated simultaneously there is a tendency for the loose ends of the ribbons to become entangled, but this is prevented by partitions.—RWY

2,335,244

6.7 STRINGED MUSICAL INSTRUMENT

Carmelo Gugino.
November 30, 1943, 3 Claims (Cl. 84–293).

There is a tendency in stringed musical instruments for the distance from strings to fingerboard to increase gradu-



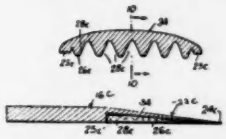
ally due to the yielding of the structure to the tension of the strings. One method of maintaining the proper relationship of the fingerboard to strings is shown in the figure as applied to a guitar. The fingerboard-neck fits somewhat loosely into the corresponding inverted T-shaped slot in the neck block **75**, so that it can pivot slightly about **84**. Screw **87** can be turned from the outside to apply a force against plate **85**, thus moving the fingerboard toward the strings.—RWY

2,342,836

6.8 REED FOR MUSICAL INSTRUMENTS

Arnold Brilhart, assignor to Arnold Brilhart Limited.
February 29, 1944, 4 Claims (Cl. 84–383).

"One object of the invention is the provision of a reed formed of a plastic and constructed so that the vibratile tongue thereof is highly flexible laterally thereof and possesses a desirable degree of stiffness along lines extending longitudinally rearwardly from the thin forward marginal edge portion of said tongue. By this construction of



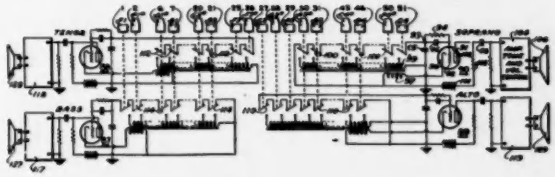
the reed, the tongue responds more readily to the control of the player of the instrument and at the same time the tone of the sounds produced by the vibration of the reed is improved in quality. Further, as the reeds are formed by molding thereof from the plastic material, it is possible to reproduce in the reed the characteristic preferred by various players which is a desirable result not easily accomplished, if at all, with wooden reeds."—RWY

2,332,076

6.9 ELECTRICAL MUSICAL INSTRUMENT

Laurens Hammond and John M. Hanert, said Hanert assignor to Hammond Instrument Company.
October 19, 1943, 3 Claims (Cl. 84–1.01).

Four complete oscillator-amplifier-loudspeaker systems are controlled by a keyboard in such a manner that four-part harmony may be produced throughout the chromatic range available to each part. In effect the keyboard is divided electrically between keys **26** and **27**, those keys above this point being used for the soprano and alto parts and those below for the tenor and bass. If two keys in the upper subdivision, for example, are depressed simultaneously the series switches **100** are so arranged that the upper of the two frequencies corresponding to the keys



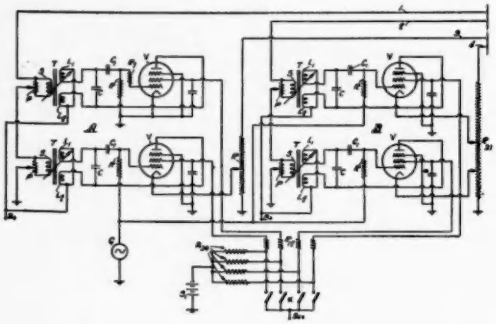
depressed is produced by the soprano oscillator. The order of the switches **110** is reversed so the lower of the two frequencies issues from the alto loudspeaker. A similar division occurs between the tenor and bass oscillators. Unisons are possible in each subdivision. For those occasions when the alto melody runs lower than the tenor the ranges of the two parts overlap by an octave centered on middle *C*. Consequently the right hand plays an octave to the right of the positions needed for the corresponding tones on a piano. Each voice is subject to individual tone and volume controls.—RWY

2,340,001

6.9 ELECTRICAL MUSICAL INSTRUMENT

Spencer W. McKellip.
January 25, 1944, 10 Claims (Cl. 84–1.12).

The tubes which are a part of the oscillators in this electrical musical instrument are biased to cut-off by S_1, but oscillations are initiated when key **K** is closed. The build-up of the voltage on the screen grid depends upon the by-pass capacitance and resistance R_{12}, so the onset and decay of sound may be modified by changes in these components. A low frequency oscillating grid bias voltage **G** (say at seven per second) produces both amplitude and frequency variations for a vibrato effect. Sounds of different qualities may be had from the output of oscillators with and without vibrato and from the cathode returns as well as from the grid circuits. Convenient means are provided for mixing these various outputs and further modifying



the wave form electrically. It is said to be an advantage when the various oscillators which are played in unison do not have quite the same frequency.—RWY

2,340,002

6.9 ELECTRICAL MUSICAL INSTRUMENT

Spencer W. McKellip and John R. Ford; said Ford assignor to said McKellip.
January 25, 1944, 14 Claims (Cl. 178–44).

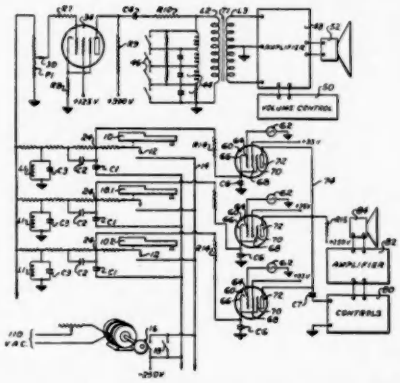
"Claim 1. In an electrical musical instrument, a source of tone frequency current including an electron tube of the high vacuum type having at least a cathode, an anode, and a control grid, an anode grid energy transfer path for said tube, means for deriving tone frequency current of one character from said path, a cathode circuit for said tube, and means for deriving tone frequency current of a different character from said cathode circuit."—RWY

2,342,338

6.9 ELECTRICAL MUSICAL INSTRUMENT

John M. Hanert, assignor to Hammond Instrument Company.
February 22, 1944, 16 Claims (Cl. 84–1.20).

In order to improve the percussive effects possible in an electrical musical instrument this invention provides an auxiliary transient tone. For example, the circuit at **18** having been completed, when contact **12** is closed condenser **C1** is caused to discharge suddenly thus momentarily exciting circuit **L1C3**. The resulting oscillating voltage is



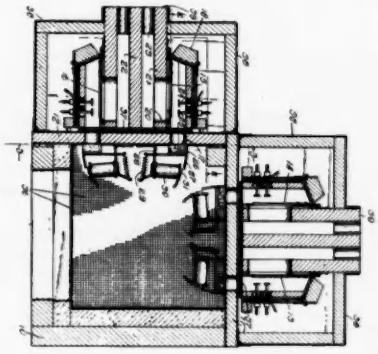
amplified by tube **34** and after further modification and amplification produces a percussive sound from speaker **52**. Moreover, the closure of contact **12** causes tube **60** to become conducting so that oscillator **62** can produce a steady sound from speaker **84** as long as the contact remains closed. The transient sound may be harmonically related in frequency to the steady one. Due to the presence of **R7** the grid current in tube **34** is limited, so the highly damped input can be made to produce an output of relatively constant amplitude for a short interval. The transient effect can be introduced periodically by the motor driven switch **16**.—RWY

2,343,728

6.9 ELECTRIC INSTRUMENT

Victor I. Zuck, assignor to Everett Piano Company.
March 7, 1944, 8 Claims (Cl. 84–1.09).

In this wind chest the air inlets are located quite close to the free ends of the vibrating reeds. This, it is claimed, improves the smoothness and quality of vibration. The arrangement is made possible by the use of an electric



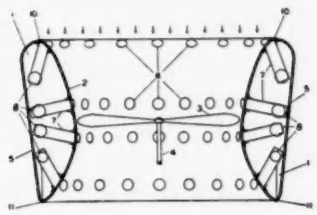
action, which is also convenient for the connection of various "stops." The design is particularly adapted to the use of electrostatic pick-ups **24** and **25**.—RWY

2,323,394

7.6 ACOUSTIC ATTENUATOR FOR AIR IMPELLERS

Eliot Huntington Harris.
July 6, 1943, 2 Claims (Cl. 181–48).

In this patent is described an acoustic attenuator which can be adapted to aeronautic or marine propellors, ventilating fans, etc. An "annuloid enclosure" is built around



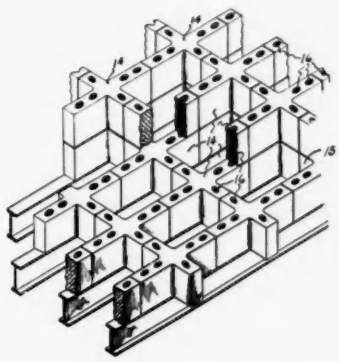
the impeller and shaped so as to maintain the most desirable flow conditions and reduce drag to a minimum. The sound is reduced by tuned resonators opening into the inner wall of the enclosure and tuned so as to reduce particularly those frequencies which are most objectionable. By locating these resonators adjacent to the source of sound, maximum sound reduction is realized.—CEN

2,324,706

7.6 METHOD AND MEANS FOR ACOUSTICAL TREATMENT

Victor Jacobson, assignor to Jacobson & Company, Incorporated.
July 20, 1943, 16 Claims (Cl. 181–35).

In this patent is described a means of attenuating extremely high noise levels by the use of long passages some



of which have "sound impeding shoulders." Other claims describe a porous material used to form various geometric designs resulting in tortuous passages. The descriptive matter refers to airplane test stands as being an application of this type of construction.—CEN

2,328,761

7.6 DOOR

Leonard B. Wamnes, Robert D. Stott, and Arthur Warren Canney.
September 7, 1943, 3 Claims (Cl. 98–87).

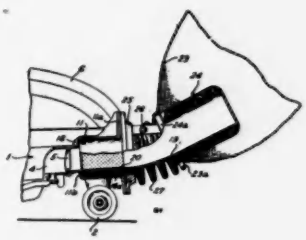
A door so constructed that a double panel section accomplishes acoustical absorption of sound yet permits free flow of air for ventilation purposes.—CEN

Re. 22,445

7.7 SUCTION CLEANER

Maurice D. Stahl, assignor to The Hoover Company.
February 22, 1944, 11 Claims (Cl. 183–36).

This muffler is designed for use on the discharge side of a suction cleaner in order to reduce the noise resulting from the rush of air. Its construction is such that it will



allow for a flexible connection between the fan chamber and the dirt bag without obstructing the flow of air. Silencing is accomplished by lining the air duct with a sound absorbent material.—CEN

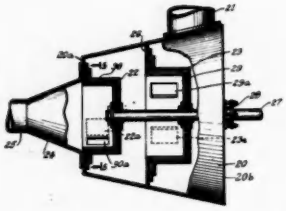
2,320,668

7.7 MECHANICAL EXHAUST SILENCER

Harry A. Smith.
June 1, 1943, 11 Claims (Cl. 181–64).

A mechanically driven valve opens and closes the exhaust system of an internal combustion engine in order

to smooth out the gas flow and so muffle the exhaust noise. It is claimed that this device, when properly timed, will



give more effective muffling with less space requirements than with conventionally designed mufflers.—CEN

2,328,161

7.7 WATER COOLED SILENCER

Hiram Hamilton Maxim, assignor to The Maxim Silencer Company.
August 31, 1943, 1 Claim (Cl. 181–52).

This wet type of spark arresting muffler was designed with the purpose of providing efficient entry of the water

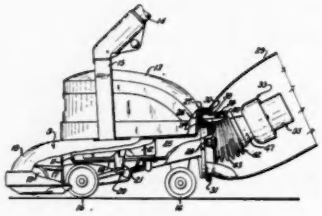
into the muffler compartment. It is claimed that the thorough mixing of exhaust gas and water assists in silencing and prevents the passage of sparks.—CEN

2,328,236

7.7 SUCTION CLEANER

Thomas A. Stoner, assignor to The Hoover Company.
August 31, 1943, 4 Claims (Cl. 183–51).

A muffler for use in a suction cleaner consists of a flexible conduit and resonant chamber in conjunction with



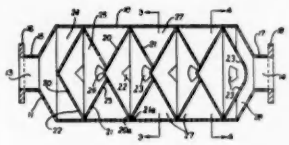
sound absorbent means. Said apparatus is contained within the dust collecting bag.—CEN

2,330,534

7.7 MUFFLER

Colinton Notman Wood.
September 28, 1943, 8 Claims (Cl. 181–57).

A muffler for use on internal combustion engines consists of a multiplicity of cones within a shell. The cones



are connected in pairs with apertures located alternately in the center and on the circumference. This causes the gases to follow a tortuous path in order to accomplish the silencing.—CEN

2,330,701

7.7 SUCTION CLEANER

Dale C. Gerber, assignor to The Hoover Company.
September 28, 1943, 9 Claims (Cl. 15–16).

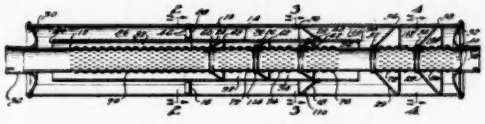
A combination resonator and absorption silencer for use on suction cleaners. This design is somewhat comparable to that shown in patent 2,328, 236.—CEN

2,331,325

7.7 METHOD OF ASSEMBLING SILENCERS

Gunnar Jensen, assignor to Walker Manufacturing Company of Wisconsin.
October 12, 1943, 3 Claims (Cl. 29–84).

This consists of apparatus designed to facilitate manufacture of the cylindrical type of exhaust muffler commonly used for automotive silencers. It is particularly designed



to improve alignments of inner tubes and shell so as to prevent the clicking noise which often accompanies the cooling of a muffler.—CEN

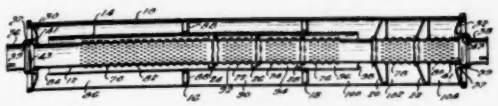
2,331,344

7.7 SILENCER

Walter H. Powers, assignor to Walker Manufacturing Company of Wisconsin.
October 12, 1943, 2 Claims (Cl. 29–84).

Designed with particular reference to use as an automobile muffler this silencer is so constructed that the inner components are able to maintain a sliding movement in

regard to the outer shell and other inner parts. It is claimed that this method of fabrication greatly reduces the amount of clicking noise which is often heard when a muffler is



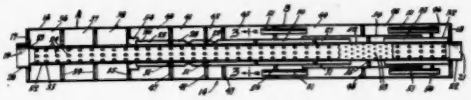
cooling down after use. It is particularly adapted to fabrication by the method described in Gunnar Jensen's patent No. 2,331,325 reviewed herewith.—CEN

2,332,543

7.7 INTERNAL COMBUSTION ENGINE EXHAUST SYSTEM

Ernest E. Wilson, assignor to General Motors Corporation.
October 26, 1943, 72 Claims (Cl. 181–48).

This design consists of a straight through type of muffler with perforated inner tubes. Various side branches



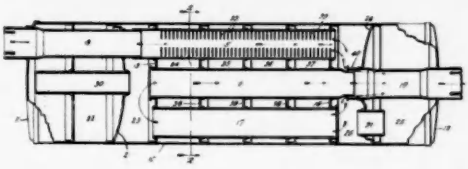
and auxiliary resonance chambers are employed in the different embodiments.—CEN

2,337,299

7.7 MUFFLER

Quintin G. Noblitt and Edmund Ludlow, assignors to Noblitt-Sparks Industries, Incorporated.
December 21, 1943, 5 Claims (Cl. 181–54).

Designed to silence internal combustion engines, this muffler combines multiple paths with side chambers for over-all silencing. Means are provided to cause interference



of the alternating pressure waves and "side silencing chambers" are included for attenuation of "a fairly wide range of frequencies."—CEN

2,337,300

7.7 MUFFLER

Quintin G. Noblitt and Edmund Ludlow, assignors to Noblitt-Sparks Industries, Incorporated.
December 21, 1943, 2 Claims (Cl. 181–54).

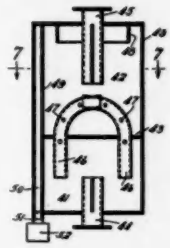
The construction shown in this patent is very similar to 2,337,299 except for the means of realizing the interference of the alternating pressures.—CEN

2,339,834

7.7 SPARK ARRESTER SILENCER

Roland B. Bourne, assignor to The Maxim Silencer Company.
January 25, 1944, 7 Claims (Cl. 181–36).

In this spark arresting muffler for use on internal combustion engines, the design shown is an improvement patent applying to 2,265,343 previously issued. The carbon



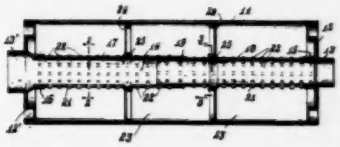
particles are so trapped as to prevent their being picked up again by the gas stream and the silencing action is improved.—CEN

2,343,152

7.7 MUFFLER

Joseph P. Marx.
February 29, 1944, 18 Claims (Cl. 181–48).

This muffler design incorporates a straight through construction with louvered perforations in the inner tube. It



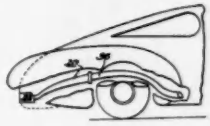
is claimed that these louvers prevent whistling noises or shell vibrations. They are also used as means of holding the inner tube by the friction of the louvers against the outer connecting tube.—CEN

2,343,371

7.7 SUCTION MUFFLER FOR AIRPLANES, AUTOMOBILES, AND THE LIKE

John A. Fisher.
March 7, 1944, 10 Claims (Cl. 181–57).

The muffler described has a long slotted opening for the inlet and means for keeping the gas flow in the form of a wide shallow stream. The outlet is located at a point of



low pressure due to movement of the vehicle. It is claimed that the result is improved muffling and reduced back-pressure.—CEN

Program of the Twenty-Ninth Meeting of the Acoustical Society of America

HOTEL PENNSYLVANIA, NEW YORK, NEW YORK

May 12–13, 1944

1. The Physics of Hearing Measurements. JOHN C. STEINBERG, *Bell Telephone Laboratories.*—This paper discusses some of the physical considerations involved in the measurement and correction of hearing impairment, and briefly reviews work which has been done in the past.

2. Articulation Tests for Hearing Aids. HALLOWELL DAVIS, *Harvard University.*

3. Methods of Testing the Electrical and Acoustical Properties of Hearing Aids. RUDOLPH NICHOLS, *Harvard University.*

4. Anatomical Changes Responsible for Blast Deafness and the Prevention of Such Damage to the Ear. STACEY R. GUILD, M.D., *Otological Laboratory, The Johns Hopkins Hospital.*

5. The Army Aural Rehabilitation Program. M. R. MOBLEY, *Lt. Col. U. S. Army, Deshon General Hospital, Butler, Pennsylvania.*

6. Acoustical Trauma as Encountered in Submarine Duty. C. W. SHILLING, *Captain (MC), U. S. Navy.*

7. Design of Non-Reverberant Rooms for Acoustic Measurements. LEO L. BERANEK, *Harvard University.*

8. Air- and Bone-Conduction Audio-Testing Assembly. NORMAN A. WATSON, *University of California at Los Angeles.*—An assembly of apparatus for the testing of hearing by air conduction and bone conduction is proposed which is designed to obviate the use of the soundproofed booth, to eliminate errors common in present techniques, and to provide more comprehensive and accurate tests of hearing than is possible with the usual audiometer. It consists of an adjustable chair and insulated chin rest; a pair of large, adjustable, acoustically-treated air-conduction source-boxes to be placed one over each ear; and an adjustable bone-conduction vibrator, mounted so as to impinge on the center of the forehead of the person being tested. If used in a quiet room, the air-conduction source-boxes would obviate the use of a soundproofed booth and insulate against air radiation from the bone-conduction vibrator without causing false enhanced bone-conduction thresholds. Air-conduction masking should always be used on one ear while testing the other by bone conduction and should be used on the better ear when testing the poorer ear by air conduction. Such an assembly, with an appropriate set of microphones, phonograph pick-up and turn table, oscillators, amplifiers, attenuators, equalizers and filters, would provide the possibility for equal-loudness as well as threshold measurements and for articulation testing with selective amplifiers by diotic as well as by monaural listening, and by various combinations of air- and bone-conduction receivers. Thus it would provide, at relatively small expense and without a soundproofed room or booth, the extensive and intensive tests of hearing for diagnosis of hearing impairment and prescription of hearing amplifiers previously available only in specialized acoustical laboratories.

9. Evidence for the Existence of Peripheral Auditory Masking. KARL LOWY, *University of Rochester.*—In the course of experiments involving the response of single auditory nerve-fibers, it was found that watch ticks elicit synchronized potentials which can be recorded from a great number of fibers. An almost identical response is easily obtained from the round window. Both can be suppressed by pure tones, a frequency around 4050 c.p.s. being most effective for a particular watch. There is experimental evidence that the tick-response, also if recorded from the fenestra rotunda, consists almost entirely of nerve potentials (aural microphonics being negligibly small in this case). Thus, it is possible to record a practically pure nerve-response from the cochlea. This opportunity was used to determine the locus of suppression of the click-potential by the masking pure tone. It is found that damage to the auditory nerve does not abolish the masking effect. One would therefore conclude that it takes place within the cochlea. This is in agreement with an assumption made by Galambos in his report on suppression of a single nerve-fiber response by a second pure tone.

10. The Consideration of Hearing Aids and Audiometers by the Council on Physical Therapy. HOWARD A. CARTER, *Council on Physical Therapy, American Medical Association.*—This paper consists of a brief account of the activities of the Council on Physical Therapy of the American Medical Association in its investigation of hearing aids and audiometers, the problems encountered in its attempt to advise on the accuracy of advertising and the methods by which evidence is considered.

11. The Role of the College and University Hearing Clinic. MIRIAM D. PAULS, *Special Education Clinics, Indiana State Teachers College.*—The introductory discussion will deal with the factors that have led to the growth of hearing conservation programs in our public schools, and the establishment of broad rehabilitation facilities for military and civilian personnel with a hearing impairment. The role of the college and the university hearing clinic will be pointed out. It would seem logical that these clinics with their broad affiliations and scientifically trained staffs can serve as the clearing house to utilize the research and facilities of the interrelated fields and thus develop sound therapeutic procedures for the rehabilitation of the acoustically handicapped. The major part of the paper will present in some detail the possible scope

of such a hearing program, by discussing the actual program of the clinic with which the speaker is affiliated. The nine clinical services that are available to the hard-of-hearing individual will be described and illustrated with pertinent case material. Implications for future growth and development in the field will be touched upon.

12. Acoustic Sound Filtration and Hearing Aids. FREDERICK M. GROSSMAN AND CHARLES T. MOLLOY, *Oto-Laryngological Division, Department of Hospitals, City of New York.*—The receiver of a hearing aid is usually connected with the ear by means of an ear mold made from a plastic material. It has been clinical experience that the dimensions of the mold canal influence the acoustical performance of hearing aids. Molds having different canal diameters are connected with a microphone in front of which is a tube approximating the external auditory canal. The other end of each mold is connected to a hearing aid receiver. The acoustical out-put of the receiver through the coupling system is registered by a high speed level recorder. The frequency response curves thus produced are found to be distinctly different for the various molds tested. The behavior of an acoustical high frequency pass filter, inserted between the receiver and the mold is also investigated. A mathematical analysis is made and sound pressures in front of normal ear drums are calculated. It is confirmed by laboratory proof that the acoustic line between receiver and drum is one of the determining factors in the performance of hearing aids. It is consequently an important consideration in the clinical fitting of hearing aids.

13. Acoustic Amplification of Hearing Aids. PAUL E. SABINE, *Riverbank Laboratories.*—This paper gives the results of measurements of the over-all acoustic gain shown by six commercial hearing aids. The input level was 50 db above the reference level of a General Radio sound level meter. The measurements covered the frequency range from 200 to 4096 c.p.s. A steady sound field was set up in an absorbent lined booth. A Western Electric 640A condenser transmitter was fitted into a hole at the center of a small baffle, with its diaphragm in the plane of the baffle face. The field was monitored by the sound level meter, with its microphone in a fixed position close to the edge of the baffle. The output of the condenser transmitter was fed into a measuring system consisting of an attenuator, amplifier, and indicating meter. The attenuation needed to produce a given reading on the meter was determined. The condenser transmitter was then replaced by the microphone of the hearing aid, with its diaphragm in the same position the transmitter diaphragm had occupied. The hearing aid receiver was coupled through a 2 cc closed coupler to the condenser transmitter and the attenuation required to give the same reading as before on the indicating meter was noted. The difference between the attenuations in the two cases was taken as a measure of the over-all acoustical gain afforded by the hearing aid. All of the instruments tested, except one, showed maximum amplification in the 1000 to 2000 c.p.s. range. Among the different instruments, this maximum was between 45 and 68 db. The one exception showed a maximum gain of 42 db at 700 c.p.s. At 300 c.p.s., the spread between the different instruments was from 12 to 55 db, while at 4096 c.p.s. it was from 18 to 52 db. Only one of the instruments tested showed any significant change in frequency response with change in the "tone control setting" of the instrument. In this instrument a change of 25 db in the amplification of tones below 512 c.p.s. was possible with a change of only 4 to 8 db at the higher frequencies.

14. Pressure and Field Response of the Ear In Hearing Aid Performance Determination. C. J. LEBEL, *The Maico Company Inc.*—Since hearing aid receiver acoustical output is commonly measured by an artificial ear wherein a microphone replaces the ear drum, a correction factor must be applied to the artificial ear out-put to relate it to the free field performance of the ear (which is the norm). In other words, substituting pressure operation for free field operation of the human ear changes its frequency response, and this change must be allowed for in determining the corrective characteristics of a hearing aid. The correction is computed and is then compared with subjective determinations. The sum of this correction and one for the baffle effect of the human body wearing the hearing aid transmitter is then computed and it is concluded that such an over-all correction is rather great. In fact, the over-all correction is so great that it cannot safely be neglected in determining or comparing hearing aid characteristics.

15. Practical Hearing Aid Measurements. R. W. CARLISLE AND A. B. MUNDEL, *Sonotone Corporation.*—The application of an inertia-controlled sound generator or artificial voice to free-field measurements of hearing aids is described. The distribution of pressure over the person, in the usual places that a hearing aid is worn, is shown, together with data on a simple baffle or artificial body which has been used to simulate the person in pressure measurements of hearing aids. A simple oscilloscopic method of simultaneously observing undue distortion while taking frequency characteristics is described. The construction of a simple artificial mastoid is described, together with its calibration. The testing of individual hearing aids in production is described. A rapidly-sweeping variable-frequency oscillator is electronically controlled and its output applied to each hearing aid. The frequency response of the instrument appears on an oscilloscope screen continuously while the volume and tone controls are varied. Acoustic coupling to the hearing aid is provided by dynamically driven sound sources, close coupled to the hearing aid and compensated to approximate speech-band free-field characteristics.

16. Introductory Remarks. L. J. SIVIAN, *Bell Telephone Laboratories.*

17. Measurements of Elastic and Dissipative Properties of Solids by Supersonic Methods. SIDNEY SIEGEL, *Westinghouse Research Laboratories.*—A number of methods used to excite mechanical vibrations of supersonic frequency in solid bars will be described. Among these are the

piezoelectric, magnetostrictive, magnetic, and eddy current methods. A large variety of investigations has been carried out with these, and the results, in fields where these methods are particularly suited, will be described. Elastic and dissipative properties of single crystals, and the variation of such properties with temperature, degree of order, magnetic state, etc. will be described.

18. High Frequency Treatment and Investigation of Organic High Polymers. H. MARK, *Polytechnic Institute of Brooklyn.*

19. The Supersonic Reflectoscope; an Instrument for Inspecting the Interior of Metal Parts by Means of Sound Waves. F. A. FIRESTONE AND J. R. FREDERICK, *University of Michigan.*—The supersonic reflectoscope sends into the metal part to be inspected a short wave train of supersonic waves. Any reflections received back at the sending point, before the reflection from the opposite side of the part, are an indication of internal flaws.

20. Application of Acoustical Methods to Mining Problems. LEONARD OBERT, *Department of Interior, Bureau of Mines.*

21. Anelasticity of Metals. CLARENCE ZENER, *Watertown Arsenal Laboratory.*—The term anelasticity is used to denote that property of metals in virtue of which strain is not a unique function of stress even at such low stress levels that no permanent set is observed. A review is given of the formal relations between the various anelastic effects, including those which may be measured by acoustical methods. A survey is made of the physical origin of anelasticity. One origin lies in the coupling of the elastic system with the various thermodynamic potentials of the system. A second basis lies in the presence of localized noncrystalline regions or interfaces. Emphasis is put upon the urgent need of a thorough study of the mechanical properties of these localized regions through acoustical studies.

22. A Fixed Path Supersonic Interferometer for Liquids. J. W. BALLOU, *Harvard University* AND J. C. HUBBARD, *The Johns Hopkins University.*—The use of the fixed path supersonic interferometer has been extended to liquids, with a view to its possible application to the measurement of sound velocities in small amounts of liquid. The present study has been confined to small cylindrical cells of fixed length, containing 5 or 6 cc of liquid, bounded at one end by a quartz vibrator having a natural frequency around 3.5 megacycles per second, and at the other by a plane reflector. Successive cell resonances were obtained by varying the frequency, and detected optically by observing the increase in diffracted light intensity which occurs when standing waves are formed. In interpreting the results, account must be taken of the departure from unity of the reflection coefficient γ, which occurs at liquid-solid boundaries. This introduces a phase change which effectively lengthens the cell path. A description of the interferometer, its theory, and the results obtained with water, dextrose solution, and ethyl alcohol are given.

23. Young's Modulus of Elasticity of Fibers and Films by Sound Velocity Measurements. J. W. BALLOU, *Harvard University* AND S. SILVERMAN, *Applied Physics Laboratory, The Johns Hopkins University.*—A method for measuring the modulus of elasticity of fibers and films by sonic means is described and the results of an exploratory study of this modulus in various natural and synthetic fibers and films are given. The method is based on the equation for the propagation of longitudinal waves in elastic materials, which expresses the velocity of propagation in terms of the elastic modulus and density of the medium. The procedure involves measuring the sound velocity and calculating the modulus, using the proper density value. The modulus so calculated is characteristic of a dynamic, short period measurement, since a sound frequency of approximately 10 kilocycles per second was used. The range of moduli found was from 1×10^{10} to 50×10^{10} dynes per square centimeter. Correlations of moduli with other physical properties are given.

24. Curved Quartz Crystals as Supersonic Generators. L. W. LABAW, *Brown University.*—The supersonic beam radiated from a one-centimeter radius quartz disk has been examined as a function of the curvature of the crystal. The large curvature crystals produce a definite focusing action as has been reported from an earlier investigation. Experimental curves show that it is possible to obtain a much greater amplitude close to the generator than is possible from a flat crystal of comparable dimensions using the same input energy. The plates of smaller curvature do not produce as marked a focusing action but give a greater supersonic amplitude than a quartz plate of zero curvature having the same thickness and excited by an equal crystal current at the same frequency, at any given distance from the generator. The beam spread for the crystals of small curvature is no greater than that for the flat crystal, indicating a transition from the wide angle to be expected in the case of a crystal which focuses the energy to the small angle predicted for the flat crystal.

25. Supersonic Interferometer as an Industrial Tool for the Study of Liquids and Solids. ELIAS KLEIN AND J. W. FITZGERALD, *Naval Research Laboratory.*

26. The Action of a Direct Radiator Loudspeaker with a Nonlinear Cone Suspension System. HARRY F. OLSON, *RCA Laboratories, Princeton, New Jersey.*—During the past few years a number of mathematical investigators have directed their efforts toward the solution of differential equations with variable coefficients. These analyses are useful in explaining some of the phenomena which occur in electroacoustic vibrating systems with nonlinear elements. In particular, this mathematics may be used to explain the various phenomena exhibited by a direct radiator loudspeaker with a nonlinear cone suspension system. One of the effects is a jump phenomenon in the response frequency characteristic. Another effect is the production of harmonics and subharmonics.

27. A Precision Method for the Determination of the Velocity of Sound in Air. R. W. LEONARD, *University of California at Los Angeles.*—A method has been developed and refined in the laboratory at U.C.L.A. for the measurement of the velocity of free progressive sound waves in air. The method involves wave-length measurements in a free sound field of known frequency. A directional source radiating "spherical" waves is placed at one end of the free field room. Measurements of positions of equal phase along the axis of the source are made with a movable microphone. The phase measurements are made on the screen of an oscilloscope. The amplified voltage developed by the microphone is applied to the vertical plates of the oscilloscope and a voltage derived from the source oscillator is applied to the horizontal plates. Closure of the ellipse on the oscilloscope screen is used to determine position of equal phase. The microphone is moved away from the source on a steel track, the position of the microphone being remotely controlled by Selsin motors. The positions of equal phase are recorded by photographing a reference scale against a steel tape stretched parallel to the axis of the source. The camera and reference scale move with the microphone. Wave-length measurements are made over an interval of 10 meters. The source frequency is determined by beating one of its harmonics with the carrier frequency of a commercial broadcasting station. Harmonics of the order of the 100th are generated in a suitable diode circuit. Temperature measurements are made by means of thermocouples distributed along the line of measurement. Humidity is determined by a gravimetric method. A sample of air of known volume is passed through a drying tube and the change in weight of the tube determined. Since it is unnecessary for the operator to enter the room during or immediately previous to making the measurements temperature gradients and turbulence are at a minimum,

28. A Theorem on Directivity Patterns. R. CLARK JONES AND L. A. MACCOLL, *Bell Telephone Laboratories.*—The directivity pattern $p(\varphi)$ of a surface distribution of acoustic sources at a single frequency and at a great distance from the source, is a function only of the angle φ between the direction and the normal to the surface, provided that the surface distribution possesses circular symmetry. Similarly the corresponding directivity pattern $q(\theta)$ of acoustic sources distributed along a line is a function of the angle θ between the direction and the plane normal to the line. Given now the complex function $A(r)$ which specifies the amplitude and phase of the circularly symmetric surface distribution, it is elementary to determine the linear distribution $B(x)$ which will have the same directivity pattern—that is to say, which will make $q(\xi)=p(\xi)$. The converse problem is also of interest: given $B(x)$, to determine $A(r)$ so that $p(\xi)=q(\xi)$. It will be shown that the solution of this converse problem involves an integral equation which may be reduced to Abel's canonical form. Accordingly, the solution may be given in closed form as the derivative of an improper integral which involves the variable of differention both in the limits and in the integrand.

29. Analyses of Tones of Wind Instruments and of Resonating Strings. F. A. SAUNDERS, *South Hadley, Massachusetts.*—In the course of studies of violins, it became of interest to find out how much their tone analyses differed from those of certain wind instruments. Analyses were made of flute, clarinet, oboe, English horn, and French horn, rather softly blown, over their whole range. The results show no such violent variations in volume of tone as in violins, though a weak spot appears in the oboe and the clarinet. For low tones in oboe, English horn, and French horn the upper partials are strongest. These die off progressively on the way up the scale until at the top the order of prominence is the order of the partials, as in high violin tones. The order of appearance of the even-numbered partials in the clarinet is interesting; 6, 4, and 2 are absent in the lowest tones, and they appear progressively, in that order. The second passes the third one octave below the top; the fundamental is nearly always loudest.

The violin family includes the *viola d'amore*, in which seven or more extra wires which are not fingered are added for resonance. Some writers and many musicians believe that the extra wires increase the loudness, though wires were once supposed to kill the sound in an auditorium. To test the matter, analyses were made of A-440 tones from each of two violins played on the D string, with or without allowing the open A string to resonate. The results show no effect unless the tuning is exact, but clearly show that the resonating string weakens the output of the fundamental without appreciably affecting the upper partials. If the energy supply given to the bowed string is constant, it is clear that the roundabout process of producing sound from the violin box by way of the resonating string must involve a loss of energy. The peculiar quality of the viola d'amore, if it exists, must be due to some cause other than the presence of the extra wires.

30. Some Problems for Post-War Musical Acoustics. ROBERT W. YOUNG, *C. G. Conn, Ltd.*—Problems of musical acoustics worthy of study may be found in at least three categories: basic studies of the vibrating media, techniques and criteria, and new devices. As an example of the first category, the velocity of sound *within* wind instruments should be investigated for various ambient conditions. The internal air temperature and humidity distribution resulting from the breath of the player are involved as well as external temperature, the openings in the instrument, its material, and its shape. This information in turn is needed for a problem in the second category, namely a criterion of standard pitch suitable for a wind band. The pitch of different wind instruments in general increases with ambient temperature, but by varying amounts. Can a "variable" standard pitch dependent upon ambient temperature be found which will permit all the instruments of a wind band to remain relatively in tune with each other under varying climatic conditions better than is possible with the nominal fixed standard now in use? In addition to new instruments, a need falling in the third category is that of a sound analyzer capable of operation during normal musical performance, but which gives the results of its analysis in a form easily comprehended and not requiring

prohibitive labor of interpretation. High speed operation is clearly indicated to delineate change of tone quality with time.

31. The Clang Tone of the Pianoforte. ARMAND F. KNOBLAUGH, *The Baldwin Company.*—This phenomenon occurs in the bass section of all pianofortes and is termed a "clang" or "wolf" tone by piano makers. It comprises a distinctly audible, high pitched sound, emitted with the bass tone when the string is struck in the usual manner. Its pitch, varying from 500 to 3000 c.p.s. throughout a piano, is constant for any one string but varies with string dimensions, being higher for the shorter, lighter bass wires. The fundamental tones of the corresponding bass strings range from 30 to 100 c.p.s. With the aid of a tone analyzer and other apparatus, the effect has been shown to be due to a longitudinal vibration of the bass string. The velocity along the string has been found to be $(AE/M)^{\frac{1}{2}}$ where A = the cross-sectional area of the core wire, E = the modulus of elasticity of the core wire material, and M = the total mass (core+wrapping) per unit length of the string. Dividing the velocity by twice the length of the string yields the frequency of the clang, confirmed by observation and experiment. The clang has its own system of partials, substantially harmonic. The component of hammer motion tangential to the string is probably an exciting cause and the complex motion of the bass bridge probably permits transfer of energy to the soundboard.

32. The Dipole Microphone. BENJAMIN OLNEY, FRANK H. SLAYMAKER, AND WILLARD F. MEEKER, *Stromberg-Carlson Company.*—A dipole microphone is defined as a microphone whose response is a function of the pressure difference between two distinct acoustic terminals. One application takes the form of a wearable, close-talking microphone in which only the acoustic dipole element need be located near the mouth of the wearer. The sound is conducted through small tubes and is applied to opposite sides of a diaphragm housed with the remainder of the microphone elements in a case attached to the head or body of the wearer. Such a microphone exhibits strong discrimination against ambient noise. Tube resonances are avoided by correct termination of the tubes at the dipole ends, and without intermediate damping. Expressions are developed for the acoustic pick-up characteristics of a dipole close to a small source; for the over-all frequency response of the microphone; and for the random energy efficiency of a dipole as compared with that of one of its poles. Measured and computed characteristics of an experimental microphone are given.

33. Sound Wave Fields Within Cavities. RICHARD K. COOK, *National Bureau of Standards.*—Cylindrical cavities of circular cross section are extensively used in acoustics in the calibration of microphones and receivers. Such a cavity having plane ends is assumed to be driven by motion of one of the plane ends. Solutions of the wave equation in the form of Fourier-Bessel expansions for the distribution of pressure and particle velocity (as a function of frequency) within the cavity are obtained. The theoretical results are compared with experimental measurements of amplitude and phase of sound pressure within cylindrical cavities.

34. Ear and Closed Coupler Acoustic Impedance. G. S. COOK AND P. CHRZANOWSKI, *National Bureau of Standards.*—The Flanders method for measurement of acoustic impedance[1] has been adapted to the measurement of ear impedance. The results of measurement of ear impedance as seen through the caps of several types of audiometer receivers are reported. These are compared with similar measurements on a standard closed coupler used for receiver calibration. The suitability of present couplers for receiver calibration is discussed.

[1] Bell Sys. Tech. J. 11, 402 (1932).

35. A Wide-Range Adjustable Acoustic Impedance. WILLARD F. MEEKER AND FRANK H. SLAYMAKER, *Stromberg-Carlson Company.*—A method for obtaining a wide range of calculable impedance values has been developed. Three telescoping tubes are used, one being acoustically "infinite" in length. This "infinite" tube is used as a convenient, calculable impedance terminating the device. The effective lengths of the remaining two sections are varied to adjust the input impedance, whose value may then be calculated in terms of that of the "infinite" section. A unit having an input section 1 inch in diameter and an "infinite" section 0.130 inch in diameter has been constructed. The range of impedance values obtainable is best described in terms of a resistance-reactance plot. On such a plot the range at 800 c.p.s. for the 1-inch diameter unit includes the impedance values enclosed by a circle having its diameter on the resistance axis and determined by the points $R=0.194$ acoustic ohm, $X=0$ and $R=342$ acoustic ohms, $X=0$. The theory is developed, and measured and calculated impedance values for several conditions are compared.

36. Propagation of Sound in Lined Ducts. CHARLES T. MOLLOY, *Material Laboratory, Navy Yard, New York, New York.*—This paper presents the theory of the propagation of sound in lined ducts, developed from a uni-dimensional wave equation. This approach was first used by Sivian. Differential equations are derived for ducts with similar linings on all walls and for ducts having different linings on each wall. A general formula is given for the attenuation constant and various special cases are discussed. Graphs are given which make the practical calculation of attenuation in ducts convenient. The variation of attenuation constant with duct size is discussed and methods for calculation of acoustical impedance from attenuation measurements given.

37. The Baffle Effect of the Human Body on the Response of a Hearing Aid. WILBUR W. HANSEN, *The Maico Company.*—Theoretical and experimental curves are presented to show the change in response of an electronic hearing aid produced when it is worn by a person. Effects produced by placing the aid in different positions on the person and by aids and persons of different sizes are shown and discussed. The effects shown are large enough to warrant their consideration in describing the response of a hearing aid.